NOTATION: For polynomials $f, g \in k[x]$ write “$f | g$” (“$f$ divides $g$”) if there is a polynomial $q \in k[x]$ such that $fq = g$.

1. Let $h, h' \in k[x]$. Write out the definitions for $h | h'$ and $h' | h$.

2. Show that $h | h'$ and $h' | h$ implies that $h$ and $h'$ only differ by a constant.

A polynomial $h \in k[x]$ is the GCD of polynomials $f, g \in k[x]$ if

- $h | f$ and $h | g$, and
- $h' | f$, and $h' | g$ implies that $h' | h$.

NOTATION: write GCD$(f, g)$ for the GCD of $f, g$.

3. Find GCD$(5x^2, x^3)$.

The Division Algorithm for single-variable polynomials says that, given $f, g \in k[x]$ ($g \neq 0$), there are unique polynomials $q, r$ such that $f = gq + r$, where either $r = 0$ or $\deg(r) < \deg(q)$.

The Euclidean Algorithm for $k[x]$ says GCD$(f, g) = \text{GCD}(g, r)$.

This statement yields an algorithm as the process of repeatedly finding $q$’s and $r$’s must eventually yield $r = 0$. The algorithm is:

(a) Use the Division Algorithm to find $q$ and $r$.
(b) If $r = 0$, STOP. The last “$g$” is the GCD.
(c) Let the current $g$ be the new “$f$”, and let the current $r$ be the new “$g$”. Repeat.
4. Use the Euclidean Algorithm to find GCD($5x^2, x^3$).

5. Use the Euclidean Algorithm to find GCD($x^2, x + 1$).