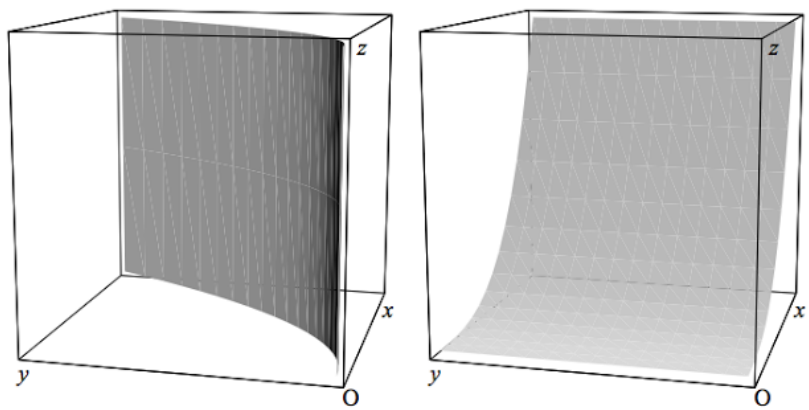
Last name _____

First name _____

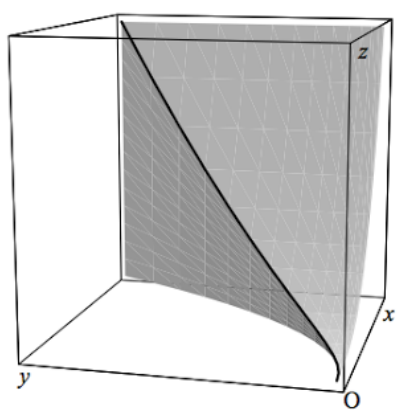
LARSON—MATH 591—CLASSROOM WORKSHEET 06
Twisted Cubic, Unions and Intersections

An interesting example of a curve in $\mathbb{R}^3$ is the *twisted cubic*, which is the variety $\mathbf{V}(y - x^2, z - x^3)$. For simplicity, we will confine ourselves to the portion that lies in the first octant. To begin, we draw the surfaces $y = x^2$ and $z = x^3$ separately:



$y = x^2$ $\qquad$ $z = x^3$

Then their intersection gives the twisted cubic:



The Twisted Cubic

Let x, y, z be polynomials in $\mathbb{R}[x, y, z]$ (so $x = x^1 y^0 z^0$, etc.)

1. Sketch $\mathbb{V}(x)$

2. Sketch $\mathbb{V}(y, x)$

3. Sketch $\mathbb{V}(x) \cup \mathbb{V}(y, x)$

4. Sketch $\mathbb{V}(xz, yz)$.