LARSON—MATH 556—SAGE WORKSHEET 10
Complements and Cliques

1. Log in to your Sage Cloud account.
   (a) Start Firefox or Chrome browser.
   (b) Go to http://cloud.sagemath.com
   (c) Click “Sign In”.
   (d) Click project Classroom Worksheets.
   (e) Click “New”, call it s10, then click “Sage Worksheet”.

The complement of a graph $G$ is a graph $\bar{G}$ with the same set of points (so $V(\bar{G}) = V(G)$) and with lines $E(\bar{G}) = \{vw : v, w \in V(\bar{G})$ and $vw \notin E(G)\}$ (that is, $vw$ is a line in $\bar{G}$ if and only if it is not a line in $G$).

2. Evaluate $\text{pete} = \text{graphs.PetersenGraph}()$. To get the complement of a graph $G$ in Sage use $G\.complement()$. So to find the complement of the Petersen graph, and to give the new graph the name $\text{pete\_complement}$ use $\text{pete\_complement} = \text{pete}\,.\text{complement}()$. Use $\text{show}()$ to draw this graph.

3. Find the size of $\text{pete\_complement}$.

4. Let $\text{k\_3\_4} = \text{graphs.CompleteBipartiteGraph}(3,4)$. Find the complement of this graph and use $\text{show}()$ to draw it.
A clique in a graph is a complete subgraph (so the points $S \subseteq V(G)$ induce a clique in graph $G$ if and only if $G$ has a line between every pair of points of $S$). A clique in a graph is maximal if it is not contained in a larger clique. A clique is maximum if it has more points than any other clique.

To find a maximum clique in a graph $G$ with Sage use `G.clique_maximum()`.

5. Find a maximum clique in the Petersen graph.

6. Find a maximum clique in the complement of the Petersen graph.

The clique number $\omega$ is the cardinality of a maximal clique.

To find the clique number of a graph $G$ in Sage, use `G.clique_number()`.

7. Find $\omega$ for the Petersen graph.

8. Find $\omega$ for the complement of the Petersen graph.

9. Find $\omega$ for $k_{3,4}$.

10. Find $\omega$ for the complement of $k_{3,4}$. 