Last name _____

First name _____

LARSON—MATH 556–SAGE WORKSHEET 05
Graph Theory Basics

1. Log in to your Sage/Cocalc account.

   (a) Start Chrome browser.

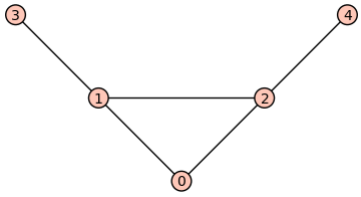
   (b) Go to `http://cocalc.com`

   (c) Click "Sign In".

   (d) Click project **Classroom Worksheets**.

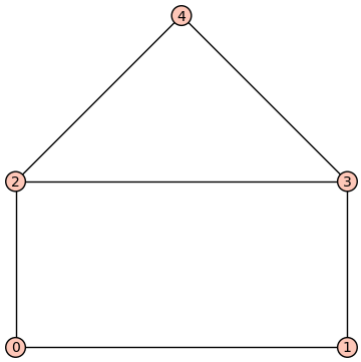
   (e) Click "New", call it **s05**, then click "Sage Worksheet".

2. Now use `Graph()`, `add_vertex()` and `add_edge()` to make the *bull*:



   Start by letting `bull=Graph(5)`. (Instead of using `add_vertex()`, you can start with `Graph(5)` to get a graph with 5 vertices and no edges.) Now add the edges that you see in the diagram of the bull using `bull.add_edge()`. Remember that the layout of the graph doesn't matter—only that it has the same edges. Write the code that worked. When you are done you can view it with `bull.show()`.

3. Make the following graph, called "the house". Start by letting `house=Graph(5)`. Write the code that worked. When you are done you can view it with `house.show()`.



Another way to represent a graph with order n is with an $n \times n$ *adjacency matrix* A. If the vertices of the graph are $\{v_0, v_1, \ldots, v_{n-1}\}$ (or {1,2,...,n-1} for short) then the $A_{i,j}$ is 1 if there is an edge from vertex i to vertex j, and 0 if there is not.

4. Evaluate:

```
g=graphs.PetersenGraph()
g.adjacency_matrix()
g.show()
```

Write the matrix. Make sure you understand the pattern of 0's and 1's.

5. One way to make a graph is to start with a number of vertices and then for each pair of vertices n and m, flip a coin to decide whether to put an edge between those vertices. `random()` gives a random number between 0 and 1. Try this:

```
g=Graph(10)
for i in [0..9]:
    for j in [0..9]:
        if i<j and random()<.5:
            g.add_edge(i,j)
g.size()
g.show()
```

6. The study of *random graphs* is huge and important and was initiated in a 1959 paper of Erdős and Renyi. Sage has a built in function to do this: `graphs.RandomGNP(n,p)`, where n is the number of vertices you want, and p is the probability of an edge ($0 \leq p \leq 1$). To simulate a coin flip, use $p = .5$. Run the following code a few times.

```
g=graphs.RandomGNP(10,.5)
g.size()
g.show()
```