LARSON—MATH 556–HOMEWORK WORKSHEET 08
Test 1 Review

You should know the following definitions, theorems, algorithms, and proofs for the test. Write out careful definitions, theorem statements, algorithms, proofs, and solutions. Turn these in at test time.

1. Give a definition for each of the following concepts **and** an example illustrating the concept.

   Definitions & Examples

   planar, incident, adjacent, isomorphic, complete graph, empty graph, bipartite graph, complete bipartite graph, k-partite graph, complete k-partite graph, complement, automorphism, subgraph, spanning subgraph, induced subgraph, degree, k-regular graph, regular graph, degree sequence, graphic sequence, walk, length, trail, path, components, connected, distance, diameter, cycle, girth, tree, acyclic, cut edge, spanning tree, edge cut, vertex cut, block, internally disjoint paths, Eulerian cycle, Eulerian graph, Hamiltonian cycle, Hamiltonian graph.

   Theorems

   Carefully state the following theorems.

   (a) The 1^{st} Theorem of Graph Theory.
   (b) The Bipartiteness Characterization Theorem.
   (c) Whitney's Theorem.
   (d) Menger's Theorem.
   (e) The Eulerian Characterization Theorem.
   (f) Dirac's Theorem.

   Notation

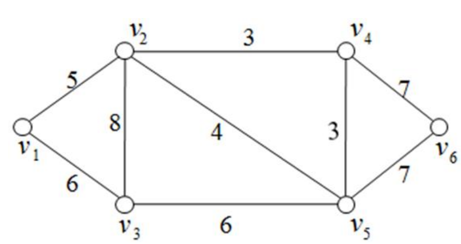
2. Give a definition for each of the following notations **and** an example illustrating its use.

   $V(G)$, $E(G)$, α, ω, κ, κ', $[S,\bar{S}]$, G^c, $G[V']$, $\nu(G)$, $\epsilon(G)$, $G \cong H$, K_n, $K_{m,n}$, $d_G(u,v)$, δ, Δ.

   Graphs

3. Draw the Petersen graph, the bow tie, K_5, K_5^c, C_5, C_5^c, P_5, and P_5^c.

   Algorithms

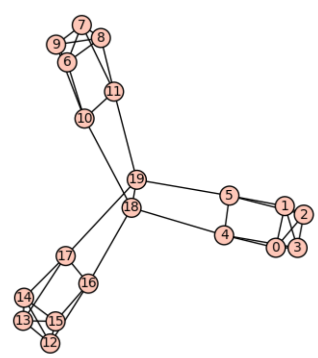


4. Use the Basic Shortest Path Algorithm to find a shortest path from v_1 to v_6. Include your steps.

5. Carefully state an algorithm for finding a maximum independent set in a graph. Explain why it works.

Proofs

6. Prove the Bipartiteness Characterization Theorem.
7. Prove: for any tree, there is a unique path between any pair of vertices.
8. Prove: If every edge in a graph is a cut edge then the graph has no cycles.
9. Prove: Every non-trivial tree has at least 2 degree-1 vertices.
10. Prove: Every connected graph has a spanning tree.
11. Prove: Every non-trivial connected graph has at least two non-cut vertices.
12. Prove: For any graph $\kappa \leq \kappa'$.
13. Prove: For any graph $\kappa' \leq \delta$.



Problems

14. Explain: the *Gould* graph does not have a Hamiltonian cycle.
15. Draw and label $K_{2,3}$. Find an adjacency matrix $\mathbf{A}$ and incidence matrix $\mathbf{M}$.
16. Explain: the Petersen graph is not bipartite.
17. Explain: if a graph has a vertex with odd degree then it does not have an Eulerian cycle (closed Eulerian trail).
18. Explain why the number of length-2 walks from vertex v_i to vertex v_j in a graph G is the i-j entry of $\mathbf{A}^2(G)$.
19. Find a minimum vertex cut of the Gould graph. Explain why it is minimum.
20. State 3 facts about trees.
21. Draw the *bow-tie* graph. Label the vertices. Find its blocks.
22. Give an example of a graph G and a subgraph H of G which is **not** induced. Explain why it is not.
23. Find an example of a graph where $\kappa < \kappa'$, an example of a graph where $\kappa' < \delta$.
24. Show that a k-regular graph of girth 5 and diameter 2 has exactly $k^2 + 1$ vertices.