1. Let the above graph be $G$. List the edges of $G$.

2. Let $E(G)$ be the vertex set of a graph $L(G)$. Draw a dot for each vertex of this new graph and label it.

3. Draw an edge between two vertices of $L(G)$ if the corresponding edges in $G$ are incident.

4. Find a maximum independent set of vertices in $L(G)$. Find $\alpha(L(G))$.

5. Check that the corresponding set of edges is independent in $G$. Check that $\alpha(L(G)) = \alpha'(G)$.

6. Find a minimum cover in $L(G)$. Find $\beta(L(G))$.

7. Check that the corresponding set of vertices is independent in $G$. Check that $\beta(L(G)) = \beta'(G)$. 

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**Gallai Identities & Brooks Theorem**

- Sec. 7.1: covering, covering number $\beta$, independent set, independence number $\alpha$, edge independence number (matching number) $\alpha'$, edge covering, edge covering number $\beta'$, Gallai Identites.

- Sec. 8.1: $k$-vertex coloring, proper (vertex) coloring, chromatic number $\chi$, $k$-chromatic-critical graph.

- Sec. 8.2: Brooks’ Theorem.
Algorithm to $\Delta$-color a non-regular graph

(a) Let the “colors” be the integers 1, 2, \ldots, $\Delta$.

(b) Choose a non-$\Delta$ vertex $v$.

(c) Partition the vertices according to their distance from $v$. Let $L_i$ be the vertices at distance $i$ from $v$.

(d) Let $k$ be the largest index. Consider the vertices in $L_k$ in any order. Color the currently considered vertex with the smallest available color. (Note that there must always be a free color until you get to $v$ itself.)

(e) Repeat for the vertices in $L_{k-1}$. And so on. Consider each successive level set with smaller index.

8. Find $\Delta$. Choose your $v$.

9. Write out your level sets $L_i$.

10. What is $k$?

11. Assign colors to the vertices in $L_k$ greedily. Repeat for the vertices in $L_{k-1}$. And so on. Consider each successive level set with smaller index.

12. Color $v$. Check that you have a proper coloring with no more than $\Delta$ colors.