Last name _____

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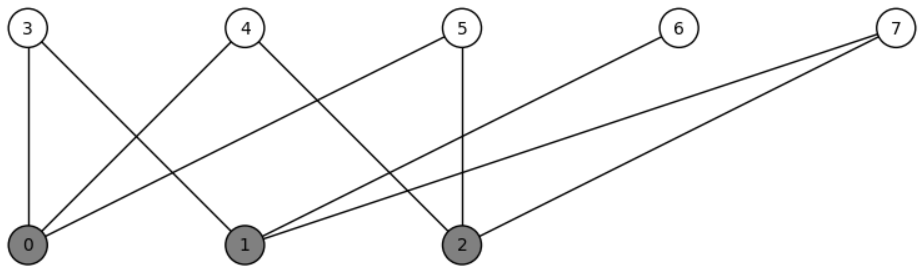
LARSON—MATH 556—CLASSROOM WORKSHEET 22
König's Edge Coloring Theorem

König's Edge Coloring Theorem says that the edges of a bipartite graph can be properly colored with Δ colors. In this example we will demonstrate an algorithm that yields a proper coloring.



1. Find Δ for the above graph G.

2. Find the set of edges E in G (we will need it later).

3. Add vertices to the black set of vertices so that there are as many black vertices as white vertices.

4. Add edges to the graph so that each vertex has degree Δ (sometimes you need to add even more vertices—but it is alsways possible). Call this graph G_1.

5. Find a perfect matching M in G_1.

6. Find $C_1 = M \cap E$ (these edges will all be colored "Color 1") in the proper edge coloring of graph G.

7. Draw $G_2 = G_1 - M$.

8. Find $\Delta(G_2)$. Check that G_2 regular.

9. Find a perfect matching M in G_2.

10. Find $C_2 = M \cap E$ (these edges will all be colored "Color 2") in the proper edge coloring of graph G.

11. Draw $G_3 = G_2 - M$. G_3 should be a matching—these edges will all be colored "Color 3".

12. Use the 3 matchings you found to properly edge color G. Make a nice drawing—with colors if possible.

The proof of König's Edge Coloring Theorem follows these steps: embed the given bipartite graph in one that is regular and with the same maximum degree. Then repeatedly identify and remove perfect matchings. These are used to provide the edge coloring guaranteed by the theorem.