Last name _____

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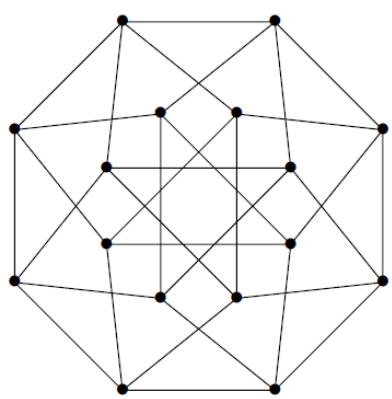
LARSON—MATH 556—CLASSROOM WORKSHEET 14
Eulerian & Hamiltonian Graphs.

Concepts & Notation

- Sec. 4.1: Eulerian cycle, Eulerian graph, Eulerian Characterization Theorem.
- Sec. 4.2: Hamiltonian cycle, Hamiltonian graph, Dirac's Theorem.

Here is an *algorithm* for finding an Eulerian cycle in a connected graph whose vertices all have even degree.

1. Pick *any* vertex. Call it v_0.
2. Go from unused edge to unused edge until you get back to v_0. Call this cycle C_0. Either you have used every edge—or there is a vertex on C_0 incident to unused edges.
3. (In the later case) Let v_1 be one of these vertices. Repeat. (Follow unused edges until you get back to v_1. Call the cycle starting at v_1, tracing C_0 then the new cycle C_1). Again, Either you have used every edge—or there is a vertex on C_1 incident to unused edges. Repeat until you get a cycle C_i with no edges adjacent to any vertex on it.

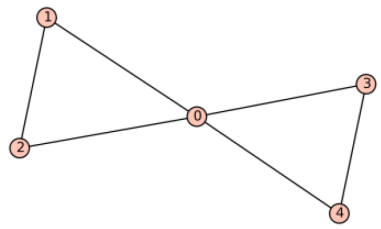


1. Use this algorithm to find a Eulerian cycle in this graph. Go slow. Label v_0, v_1, etc. After you have C_0, use a different color or shading for the next iteration.

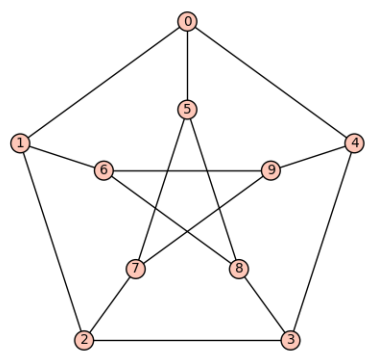
A cycle which includes every vertex of a graph G is called a **Hamilton cycle** of G. (It necessarily has no repeated edges, and only the starting and ending vertex are repeated.)

2. Draw the complete graph K_5. Find one Hamilton cycle in K_5.

3. Does every complete graph have a Hamilton cycle? Explain.



4. The bow tie graph does not have a Hamilton cycle. Explain.



5. Does the Petersen graph have a Hamilton cycle?