Eulerian & Hamiltonian Graphs.

- Sec. 4.1: Eulerian cycle, Eulerian graph, Eulerian Characterization Theorem.
- Sec. 4.2: Hamiltonian cycle, Hamiltonian graph, Dirac’s Theorem.

Here is an algorithm for finding an Eulerian cycle in a connected graph whose vertices all have even degree.

1. Pick any vertex. Call it $v_0$.
2. Go from unused edge to unused edge until you get back to $v_0$. Call this cycle $C_0$. Either you have used every edge—or there is a vertex on $C_0$ incident to unused edges.
3. (In the later case) Let $v_1$ be one of these vertices. Repeat. (Follow unused edges until you get back to $v_1$. Call the cycle starting at $v_1$, tracing $C_0$ then the new cycle $C_1$). Again, Either you have used every edge—or there is a vertex on $C_1$ incident to unused edges. Repeat until you get a cycle $C_i$ with no edges adjacent to any vertex on it.

1. Use this algorithm to find a Eulerian cycle in this graph. Go slow. Label $v_0$, $v_1$, etc. After you have $C_0$, use a different color or shading for the next iteration.
A cycle which includes every vertex of a graph $G$ is called a **Hamilton cycle** of $G$. (It necessarily has no repeated edges, and only the starting and ending vertex are repeated.)

2. Draw the complete graph $K_5$. Find one Hamilton cycle in $K_5$.

3. Does every complete graph have a Hamilton cycle? Explain.

![Complete graph $K_5$](image)

4. The bow tie graph does not have a Hamilton cycle. Explain.

![Bow tie graph](image)

5. Does the Petersen graph have a Hamilton cycle?