Last name _____

First name _____

LARSON—MATH 525—CLASSROOM WORKSHEET 32

Burnside's Lemma—Cycle Decompositions

1. Now apply theory to find the number $\mathcal{N}$ of non-equivalent colorings of the corners of a square (regular 4-gon) with the colors red, green and blue ("R", "G" and "B").

   (a) What is the symmetry group G of the square?

   (b) For each permutation $f \in G$ find $(C)(f)$ (or, more importantly, find $|\mathcal{C}(f)|$).

   (c) Apply Burnside's Lemma to find $\mathcal{N}$. Burnside's Lemma says:

$$\mathcal{N} = \frac{1}{|G|} \sum_{f \in G} |\mathcal{C}(f)|$$

2. Find the cycle decompositions for all 6 permutations in the symmetry group (D_3) of an equilateral triangle.

3. For each $f \in D_3$ determine $|\mathcal{C}(f)|$, assuming there are 2 colors to color the corners of the triangle.

4. For each $f \in D_3$ determine $|\mathcal{C}(f)|$, assuming there are 3 colors to color the corners of the triangle.

5. For each $f \in D_3$ determine $|\mathcal{C}(f)|$, assuming there are p colors to color the corners of the triangle.