Last name ______________________

First name ______________________

LARSON—MATH 401—SAGE WORKSHEET 06
Polynomial Rings & Finite Fields.

1. Log in to your Sage/Cocalc account.

   (a) Start Chrome browser.
   (b) Go to `http://cocalc.com`
   (c) Click "Sign In".
   (d) Click project **Math 401**.
   (e) Click "New", call it **s06**, then click "Sage Worksheet".

   Write Sage's responses.

2. First construct the group $\mathbb{Z}_3$. Evaluate: `Z3 = Integers(3)`.

3. Now list the elements of Z_3. Evaluate: `Z3.list()`.

4. To get the Cayley table for addition in $\mathbb{Z}_3$,
   evaluate: `Z3.addition_table(names="elements")`

5. You can also get the multiplication table for $\mathbb{Z}_3$.
   Evaluate: `Z3.multiplication_table(names="elements")`. Can you tell from the table whether every element of $\mathbb{Z}_3$ has a multiplicative inverse?

6. Is $\mathbb{Z}_3$ a field? Evaluate: `Z3.is_field()`

7. Now lets make the ring of polynomials over $\mathbb{Z}_3$ with indeterminate 'x'. Let's call it `Z3polys`. Evaluate: `Z3polys.<x> = Z3[]`. Then evaluate `Z3polys` to see what that object is.

8. Sage starts out by assuming x is a real-number variable. Let's see what it thinks now. Evaluate: `x.parent()`.

9. Is this polynomial ring finite? Evaluate: `Z3polys.is_finite()`.

10. Check if it is a field. Is it an integral domain? Evaluate `Z3polys.is_integral_domain()`.

11. Let's define a our favorite polynomial. Evaluate: `p = x**2+1`. And then `p.parent()`.

12. Is p irreducible in `Z3polys`? Evaluate: `p.is_irreducible()`.

13. Let's make a polynomial that's not irreducible. Evaluate: `f = (x+1)*(x-1)`, then evaluate `f`, then evaluate `f.is_irreducible()`. So now try `f.factor()`.

14. Now let's make the ideal generated by p in `Z3polys`. Evaluate: `id = Z3polys.ideal(p)`. We called it *id*. Evaluate `id` to see what Sage says about this object.

15. Is it principal? Evaluate: `id.is_principal()`.

16. Now lets create the quotient ring $Q = \mathbb{Z}_3[x]/\langle x^2+1\rangle$. Evaluate: `Q = Z3polys.quotient(p)`.

17. We proved this is a field. Check by evaluating: `Q.is_field()`.

18. How many elements does it have (what is Q's *order*?). Evaluate: `Q.order()`.

19. Let's make a Cayley table for addition: `Q.addition_table(names="elements")`. x is an element of `Z3polys`; its just a representative of an equivalence class in Q. Sage uses $\bar{x}$ (`xbar`) as a generic element here. Write this out using $\bar{x}$.

20. Now make the Cayley table for multiplication. Write it out here using $\bar{x}$.

21. Write each element and find its inverse.