Last name _____

First name _____

LARSON—MATH 401—SAGE WORKSHEET 05
$\mathbb{Z}_n$, the Integers mod n.

1. Log in to your Sage/Cocalc account.

   (a) Start Chrome browser.

   (b) Go to `http://cocalc.com`

   (c) Click "Sign In".

   (d) Click project **Math 401**.

   (e) Click "New", call it **s05**, then click "Sage Worksheet".

   Write Sage's responses.

2. First construct the group $\mathbb{Z}_5$. Evaluate: `Z5 = Integers(5)`.

3. Now list the elements of Z_5. Evaluate: `Z5.list()`.

4. Evaluate: `a=Z5(3)` to define an element a which is the "3" in $\mathbb{Z}_{\not\triangleq}$.

5. To check what algebraic structure Sage thinks a belongs to, evaluate: `a.parent()`.

6. To test a for membership in $\mathbb{Z}_5$, evaluate: `a in Z5`.

7. To get the Cayley table for addition in $\mathbb{Z}_5$,
   evaluate: `Z5.addition_table(names="elements")`

8. We know that $\mathbb{Z}_5$ is actually a partition of the integers. So *every* integer is in one of these partitions. Which partition is -1 in? Evaluate: `Z5(-1)` to get the canonical equivalence class representative (the remainder from the Division Algorithm).

9. Find the canonical equivalence class representative for 123 in $\mathbb{Z}_5$. What command will you use?

10. You can also get the multiplication table for $\mathbb{Z}_5$.
    Evaluate: `Z5.multiplication_table(names="elements")`. Can you tell from the table whether every element of $\mathbb{Z}_5$ has a multiplicative inverse?

11. Is $\mathbb{Z}_5$ a field? Evaluate: `Z5.is_field()`

12. Define the additive group $\mathbb{Z}_4$ of integers mod 4 in Sage. What command will you use?

13. Find the canonical representative for 123 in $\mathbb{Z}_4$.

14. Find the multiplication table for $\mathbb{Z}_4$.

15. Is $\mathbb{Z}_4$ a field? What command will you evaluate to check?