Last name ______________________

First name ______________________

LARSON—MATH 401—h12.
Test 3 Review

Turn in a nicely written-up test review at test time.

Definitions Give an example for each concept.

1. ring (ring axioms)
2. commutative ring
3. ring with identity
4. zero divisors in a ring
5. integral domain
6. division ring
7. field
8. finite field
9. ring homomorphism
10. ideal
11. subring
12. proper ideal
13. maximal ideal
14. polynomial ring $R[x]$.
15. degree of $p(x) \in R[x]$.
16. reducible polynomial in $R[x]$.
17. irreducible polynomial in $R[x]$.
18. zero of $p(x) \in R[x]$.

Theorems. State each theorem carefully and precisely.

19. 3 Properties of Rings.
20. Wedderburn's Little Theorem.
21. Maximal Ideals–Fields Theorem.
22. 1^{st} Isomorphism Theorem for Rings.

23. Division Algorithm for Polynomials.

24. Factors–Zeros Theorem.

Problems. Talk through your work (write what you're thinking). Its a good habit.

25. Let $\phi : G \to H$ be a group homomorphism. Show $\phi(G)$ is a group (and so a subgroup of H).

26. Show that if G is abelian then $\phi(G)$ is also abelian.

27. Show that if $\ker \phi = \{e\}$ (where e is the identity element in G) then ϕ is one-to-one (injective).

28. Show that, if $\phi : R_1 \to R_2$ is a ring homomorphism, then $\ker \phi$ is an ideal.

29. Let R be a ring and $a \in R$. Show: $0a = 0$.

30. Give an example of a ring that is an integral domain but not a field. Explain.

31. Give an example of a ring that is commutative with identity but not an integral domain. Explain.

32. Show: every field is an integral domain.

33. Suppose R is an integral domain. Show: for every $a, b, c \in R$, $a \neq 0$, if $ab = ac$ then $b = c$.

34. Give an example of a ring R with an identity and a non-trivial subring S without identity.

35. Give an example of a proper ideal in $\mathbb{Z}$.

36. We proved that $\mathbb{Z}_n$ is a field when n is prime. Why does $\mathbb{Z}_n$ fail to be a field for composite n?

37. Find an example of an ideal that's *not* maximal in the ring $\mathbb{Z}$.

38. We showed that $5\mathbb{Z}$ is a maximal in the ring $\mathbb{Z}$. What does the *Maximal Ideals—Fields Theorem* say in this case?

39. What is $\mathbb{Z}_2$? Define an addition and multiplication that makes $(\mathbb{Z}_2, +\cdot)$ a field.

40. What is $\mathbb{Z}_2[x]$ (and give some examples of specific elements)?

41. Find an example of an irreducible polynomial $f(x)$ of the form $x^2 + bx + c$ in $\mathbb{Z}_2[x]$.

42. Explain why $\mathbb{Z}_2[x]/\langle f(x) \rangle$ is a field.

43. What are the elements of $\mathbb{Z}_2[x]/\langle f(x) \rangle$?

44. What is the order of $\mathbb{Z}_2[x]/\langle f(x) \rangle$?

45. For each element in $\mathbb{Z}_2[x]/\langle f(x) \rangle$, find its multiplicative inverse. Explain.