Last name _____

First name _____

LARSON—MATH 401—Homework 10.

1. FOIL ("firsts, outsides, insides, last"). Show, using only the ring axioms, that for any ring R and elements a, b, c, d that

$$(a + b) \cdot (c + d) = (ac + ad + bc + bd).$$

2. Consider $(\mathbb{Z}_n, +, \cdot)$ as a ring. Define the "addition" and the "multiplication".

3. Show that $(\mathbb{Z}_n, +, \cdot)$ satisfies the ring axioms.

4. What *special* (additional) properties does this ring have (beyond the ones that come from the group axioms). Explain.

5. Why isn't $(\mathbb{Z}_6, +, \cdot)$ an integral domain? Explain.

6. Show that $\mathbb{Z}_n$ is an integral domain if and only if n is prime.

Let $(R_1, +_1, \cdot_1)$ and $(R_2, +_2, \cdot_2)$ be rings. A *homomorphism* from R_1 to R_2 is a function which preserves the ring operations. Formally, this means there is a function $\phi : R_1 \to R_2$ such that:

$$\text{For any } r, r' \in R_1 \;\; \phi(r +_1 r') = \phi(r) +_2 \phi(r'), \text{and}$$

$$\text{For any } r, r' \in R_1 \;\; \phi(r \cdot_1 r') = \phi(r) \cdot_2 \phi(r').$$

An *ideal* of a ring R is a subset I that is closed with respect to addition and, for every $x \in I$ and $r \in R$, both $rx, xr \in I$ (closed with respect to left and right multiplication by elements of R).

7. Show that, if $\phi : R_1 \to R_2$ is a ring homomorphism, then $\ker \phi$ is an ideal.