Last name _____

First name _____

LARSON—MATH 401—h08.
Test 2 Review

Turn in a nicely written-up test review at test time.

Definitions

1. $\mathbb{Z}$.
2. $\mathbb{Z}_n$.
3. $n\mathbb{Z}$.
4. $\mathbb{Z}/n\mathbb{Z}$.
5. subgroup of a group.
6. Group Axioms.
7. the cosets of a subgroup H of a group G.
8. G/H, for a group G and a subgroup H.
9. $[G : H]$.
10. isomorphism.
11. homomorphism.
12. the *kernel*, $\ker \phi$, of a homomorphism ϕ.

Theorems

13. Properties of Groups.
14. Lagrange's Theorem.
15. 1st Isomorphism Theorem.

Problems

16. Define an "addition" $+$ for $\mathbb{Z}_n$ and show that $(\mathbb{Z}_n, +)$ is a group.
17. Use the definitions to find $\mathbb{Z}/5\mathbb{Z}$.
18. Explain that the sets you found here are *exactly* the same sets as the ones in $\mathbb{Z}_5$.

The Binomial Formula is:

$$(a+b)^p = \sum_{k=0}^{p} \binom{p}{k} a^k b^{(p-k)}$$

19. Use the Binomial Formula to find $(a+b)^7$ for $a, b \in \mathbb{Z}_7$. Explain.
20. Show that any subgroup H of a commutative group G is commutative.
21. Find all the subgroups of $\mathbb{Z}_5$.
22. Find all the subgroups of $\mathbb{Z}_6$.
23. If $(G, +)$ is a group with the property that for every $g \in G$, $g + g = e$, show that G is commutative.
24. For a group G and subgroup H, what is a sufficient condition for G/H to be a group?
25. For a group G and subgroup H, show that the cosets of H in G have the same cardinality.
26. Show that $\mathbb{Z}$ and $5\mathbb{Z}$ are isomorphic.
27. Give an example of a homomorphism that is *not* an isomorphism. Explain.