LARSON—MATH 401—h08.
Test 2 Review

Turn in a nicely written-up test review at test time.

Definitions

1. \( \mathbb{Z} \).
2. \( \mathbb{Z}_n \).
3. \( n\mathbb{Z} \).
4. \( \mathbb{Z}/n\mathbb{Z} \).
5. subgroup of a group.
7. the cosets of a subgroup \( H \) of a group \( G \).
8. \( G/H \), for a group \( G \) and a subgroup \( H \).
9. \( [G : H] \).
10. isomorphism.
11. homomorphism.
12. the kernel, \( \ker \phi \), of a homomorphism \( \phi \).

Theorems

14. Lagrange’s Theorem.
15. 1\( ^{st} \) Isomorphism Theorem.

Problems

16. Define an “addition” + for \( \mathbb{Z}_n \) and show that \( (\mathbb{Z}_n,+) \) is a group.
17. Use the definitions to find \( \mathbb{Z}/5\mathbb{Z} \).
18. Explain that the sets you found here are exactly the same sets as the ones in \( \mathbb{Z}_5 \).
The Binomial Formula is:

\[(a + b)^p = \sum_{k=0}^{p} \binom{p}{k} a^k b^{p-k}\]

19. Use the Binomial Formula to find \((a + b)^7\) for \(a, b \in \mathbb{Z}_7\). Explain.

20. Show that any subgroup \(H\) of a commutative group \(G\) is commutative.

21. Find all the subgroups of \(\mathbb{Z}_5\).

22. Find all the subgroups of \(\mathbb{Z}_6\).

23. If \((G, +)\) is a group with the property that for every \(g \in G\), \(g + g = e\), show that \(G\) is commutative.

24. For a group \(G\) and subgroup \(H\), what is a sufficient condition for \(G/H\) to be a group?

25. For a group \(G\) and subgroup \(H\), show that the cosets of \(H\) in \(G\) have the same cardinality.

26. Show that \(\mathbb{Z}\) and \(5\mathbb{Z}\) are isomorphic.

27. Give an example of a homomorphism that is \textit{not} an isomorphism. Explain.