Definitions

1. What are the natural numbers $\mathbb{N}$ according to our book (Judson)?

2. State the 8 field axioms for a set $X$ with two closed operations “+” (addition) and “·” (multiplication) on $X$.

3. Cartesian Product $X \times Y$.

4. Relation on $X$.

5. Equivalence relation.

6. Equivalence classes.

7. Partition.

8. One-to-one function.


10. Bijection (bijective function).

11. gcd

12. Prime integer.

13. Relatively prime integers.

Theorems

14. The Equivalence Relations-Partitions Theorem.

15. The Division Algorithm.

16. The GCD Theorem.

17. Euclid’s Lemma.

Problems

18. State 3 common examples of fields.

19. Explain why the integers with ordinary addition are not a field.
20. True or False: 3|111? Explain.
21. True or False: 51 is prime? Explain.
22. True or False: 5 \equiv 25 \mod 4? Explain.
23. Show that congruence mod 6 defines an equivalence relation on \( \mathbb{Z} \).
   Let \( a \sim b \) if \( a \equiv b \) mod 6.
24. Find all the distinct (meaning different, or non-equal) equivalence classes for this relation.
25. For functions \( f : A \to B \) and \( g : B \to C \), if \( f \) and \( g \) are one-to-one show that \( g \circ f \) is one-to-one.
26. Let \( a \in \mathbb{Z}, b \in \mathbb{N} \) and \( S = \{a-bk : a-bk \geq 0 \text{ and } k \in \mathbb{Z}\} \). Argue that \( S \) is non-empty.
27. Show that \( \gcd(a, b) = \gcd(-a, -b) \), for integers \( a, b \) not both 0.
28. Find \( \gcd(0, 47) \). Explain.
29. Argue that \( \gcd(n, n + 1) = 1 \).
30. Find integers \( k, l \) such that \( \gcd(30, 31) = 30k + 31l \).
31. Let \( a \) be an integer and \( b \) a natural number, and \( S = \{ak + bl : k, l \in \mathbb{Z}, ak + bl > 0\} \).
   Argue that \( S \) is non-empty.
32. Let \( n \) be a natural number. Use the division algorithm to prove that every integer is congruent \( \mod n \) to precisely one of the integers 0, 1, 2, \ldots, n – 1.