Last name _____

First name _____

LARSON—MATH 401—CLASSROOM WORKSHEET 28
Ideals & Ring Homomorhisms.

A *ring* R is a set with 2 closed binary relations satisfying certain properties. The operations are usually written "+" and "·", as a ring is a generalization of the integers $(\mathbb{Z}, +, \cdot)$:

1. $(R, +)$ is an abelian group: so there is an additive identity 0, and additive inverses.
2. $(R, \cdot)$ is associative.
3. Distributive properties: for every $a, b, c \in R$ $(a + b) \cdot c = a \cdot c + b \cdot c$, and $a \cdot (b + c) = a \cdot b + a \cdot c$.

Other properties that a ring R might have include:

1. **commutative** - if the multiplication is commutative.
2. **with identity** - if there is a multiplicative identity (or "1").
3. **no zero divisors** - if $ab = 0$ implies $a = 0$ or $b = 0$.
4. **integral domain** - if commutative with identity and no zero divisors.
5. **division ring** - if every non-zero element has a multiplicative inverse (and hence (with multiplicative identity).
6. **field** - if a commutative division ring (with multiplicative identity).

We proved:

Proposition: every field is an integral domain.

Cancellation Law: If R is a commutative ring with identity, R is an integral domain if and only if for every $a, b, c \in R$, $a \neq 0$, $ab = ac$ implies $b = c$.

Wedderburn's Little Theorem: Every finite integral domain is a field.

A *subring* in a ring $(R, +, \cdot)$ is a subset S that is a ring with the inherited operations: $(S, +\cdot)$ is a ring.

An *ideal* of a ring R is a subset I that is closed with respect to addition and, for every $x \in I$ and $r \in R$, both $rx, xr \in I$ (closed with respect to left and right multiplication by elements of R).

1. Show that $5\mathbb{Z}$ is an ideal of $\mathbb{Z}$.

2. Show, for any ring with identity R and ideal I, that I is a subring of R.