A ring $R$ is a set with 2 closed binary relations satisfying certain properties. The operations are usually written “$+$” and “$\cdot$”, as a ring is a generalization of the integers $(\mathbb{Z}, +, \cdot)$:

1. $(R, +)$ is an abelian group: so there is an additive identity 0, and additive inverses.
2. $(R, \cdot)$ is associative.
3. Distributive properties: for every $a, b, c \in R$ $(a + b) \cdot c = a \cdot c + b \cdot c$, and $a \cdot (b + c) = a \cdot b + a \cdot c$.

Other properties that a ring $R$ might have include:

1. **commutative** - if the multiplication is commutative.
2. **with identity** - if there is a multiplicative identity (or “1”).
3. **no zero divisors** - if $ab = 0$ implies $a = 0$ or $b = 0$.
4. **integral domain** - if commutative with identity and no zero divisors.
5. **division ring** - if every non-zero element has a multiplicative inverse (and hence (with multiplicative identity).
6. **field** - if a commutative division ring (with multiplicative identity).

We proved:

**Proposition**: every field is an integral domain.

**Cancellation Law**: If $R$ is a commutative ring with identity, $R$ is an integral domain if and only if for every $a, b, c \in R$, $a \neq 0$, $ab = ac$ implies $b = c$.

**Wedderburn’s Little Theorem**: Every finite integral domain is a field.

A **subring** in a ring $(R, +, \cdot)$ is a subset $S$ that is a ring with the inherited operations: $(S, +, \cdot)$ is a ring.

An **ideal** of a ring $R$ is a subset $I$ that is closed with respect to addition and, for every $x \in I$ and $r \in R$, both $rx, xr \in I$ (closed with respect to left and right multiplication by elements of $R$).
1. Show that \(5\mathbb{Z}\) is an ideal of \(\mathbb{Z}\).

2. Show, for any ring with identity \(R\) and ideal \(I\), that \(I\) is a subring of \(R\).