LARSON—MATH 401—CLASSROOM WORKSHEET 25

Rings.

A ring $R$ is a set with 2 closed binary relations satisfying certain properties. The operations are usually written “$+$” and “$\cdot$”, as a ring is a generalization of the integers $(\mathbb{Z}, +, \cdot)$:

1. $(R, +)$ is a group: so there is an additive identity $0$, and additive inverses.

2. Distributive properties: for every $a, b, c \in R$ $(a + b) \cdot c = a \cdot c + b \cdot c$, and $a \cdot (b + c) = a \cdot b + a \cdot c$.

$\mathbb{Z}$ and $\mathbb{Z}_n$ are our paradigm examples of rings.

1. Explain why $\mathbb{Z}_n$ is a ring.

2. Let $R$ be a ring and $a \in R$. Show: $0a = 0$. 