Last name _____

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LARSON—MATH 401—CLASSROOM WORKSHEET 24
1st Isomorphism Theorem.

1st **Isomorphism Theorem.** If G is an abelian group, H is any group, and $\phi : G \to H$ is a homomorphism, then $G/\ker\phi \cong \phi(G)$.

(Here, $\phi(G) = \{\phi(g) : g \in G\}$, and if ϕ is onto—so $\phi(G) = H$—then $G/\ker\phi \cong H$).

We'll investigate what this says for $\phi : \mathbb{Z} \to \mathbb{Z}_5$ with $\phi(k) = [k]$.

1. Find $\ker\phi$.

2. Find $\phi(\mathbb{Z})$.

3. Find $\mathbb{Z}/\ker\phi$.

4. Check that $\mathbb{Z}/\ker\phi \cong \phi(\mathbb{Z})$.

A *ring* R is a set with 2 closed binary relations satisfying certain properties. The operations are usually written "+" and "$\cdot$", as a ring is a generalization of the integers $(\mathbb{Z}, +, \cdot)$:

(a) $(R, +)$ is an abelian group: so there is an additive identity 0, and additive inverses.

(b) $(R, \cdot)$ is associative.

(c) Distributive properties: for every $a, b, c \in R$ $(a + b) \cdot c = a \cdot c + b \cdot c$, and $a \cdot (b + c) = a \cdot b + a \cdot c$.

$\mathbb{Z}$ and $\mathbb{Z}_n$ are our paradigm examples of rings.

5. Explain why $\mathbb{Z}_n$ is a ring.