1st Isomorphism Theorem. If $G$ is an abelian group, $H$ is any group, and $\phi : G \to H$ is a homomorphism, then $G/\ker \phi \cong \phi(G)$.

(Here, $\phi(G) = \{\phi(g) : g \in G\}$, and if $\phi$ is onto—so $\phi(G) = H$—then $G/\ker \phi \cong H$).

We’ll investigate what this says for $\phi : \mathbb{Z} \to \mathbb{Z}_5$ with $\phi(k) = [k]$.

1. Find $\ker \phi$.

2. Find $\phi(\mathbb{Z})$.

3. Find $\mathbb{Z}/\ker \phi$.

4. Check that $\mathbb{Z}/\ker \phi \cong \phi(\mathbb{Z})$. 
A *ring* $R$ is a set with 2 closed binary relations satisfying certain properties. The operations are usually written “+” and “·”, as a ring is a generalization of the integers $(\mathbb{Z}, +, \cdot)$:

(a) $(R, +)$ is an abelian group: so there is an additive identity 0, and additive inverses.

(b) $(R, \cdot)$ is associative.

(c) Distributive properties: for every $a, b, c \in R$ $(a + b) \cdot c = a \cdot c + b \cdot c$, and $a \cdot (b + c) = a \cdot b + a \cdot c$.

$\mathbb{Z}$ and $\mathbb{Z}_n$ are our paradigm examples of rings.

5. Explain why $\mathbb{Z}_n$ is a ring.