**Last name** _____

**First name** _____

**LARSON—MATH 401—CLASSROOM WORKSHEET 22**
**Group Homomorphisms.**

Let $(G_1, +_1)$ and $(G_2, +_2)$ be groups.

A *homomorphism* from $G_1$ to $G_2$ if there is a function which preserves the group operation. Formally, this means there is a function $\phi : G_1 \to G_2$ such that:

$$\text{For any } g, g' \in G_1 \;\; \phi(g +_1 g') = \phi(g) +_2 \phi(g').$$

**Importantly, this map doesn't need to be 1-1. Some elements may have the same functional value.**

Let $e_1$ be the identity element in $G_1$ and $e_2$ be the identity element in $G_2$.
We will show that for *any* homomorphism $\phi : G_1 \to G_2$, we have $\phi(e_1) = e_2$.

1. Consider $\phi(e_1 +_1 e_1)$. Right what the homomorphism condition means here.

2. Find $e_1 +_1 e_1$.

3. Use that to rewrite the homomorphism condition.

4. You should have an equation of the form: $b = b +_2 b$, for some $b \in G_2$. $G_2$ is a group. So what useful thing must be true about $b$?

5. Use that fact to solve for $b$.

6. What can you conclude about $\phi(e_1)$?