Last name _____

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LARSON—MATH 401—CLASSROOM WORKSHEET 21
Group Homomorphisms.

Let $(G_1, +_1)$ and $(G_2, +_2)$ be groups.

A *homomorphism* from G_1 to G_2 if there is a function which preserves the group operation. Formally, this means there is a function $\phi : G_1 \to G_2$ such that:

$$\text{For any } g, g' \in G_1 \;\; \phi(g +_1 g') = \phi(g) +_2 \phi(g').$$

Importantly, this map doesn't need to be 1-1. Some elements may have the same functional value.

We will now show that there is always at least one homomorphism from one group to another, one where you map every element from the first group to the identity element of the second group. Let e_2 be the identity element in G_2.

Formally, define a function $\phi : G_1 \to G_2$ such that:

$$\text{For } g \in G_1 \;\; \phi(g) = e_2.$$

1. Check that ϕ is a homomorphism.

   (a) Let $g, g' \in G_1$. Find $\phi(g +_1 g')$.

   (b) Find $\phi(g)$.

   (c) Find $\phi(g')$.

   (d) Find $\phi(g) +_2 \phi(g')$.

   (e) Check that $\phi(g +_1 g') = \phi(g) +_2 \phi(g')$.