Let \((G_1, +_1)\) and \((G_2, +_2)\) be groups.

A homomorphism from \(G_1\) to \(G_2\) if there is a function which preserves the group operation. Formally, this means there is a function \(\phi : G_1 \to G_2\) such that:

\[
\text{For any } g, g' \in G_1 \quad \phi(g +_1 g') = \phi(g) +_2 \phi(g').
\]

Importantly, this map doesn’t need to be 1-1. Some elements may have the same functional value.

We will now show that there is always at least one homomorphism from one group to another, one where you map every element from the first group to the identity element of the second group. Let \(e_2\) be the identity element in \(G_2\).

Formally, define a function \(\phi : G_1 \to G_2\) such that:

\[
\text{For } g \in G_1 \quad \phi(g) = e_2.
\]

1. Check that \(\phi\) is a homomorphism.

   (a) Let \(g, g' \in G_1\). Find \(\phi(g +_1 g')\).

   (b) Find \(\phi(g)\).

   (c) Find \(\phi(g')\).

   (d) Find \(\phi(g) +_2 \phi(g')\).

   (e) Check that \(\phi(g +_1 g') = \phi(g) +_2 \phi(g')\).