LARSON—MATH 401—CLASSROOM WORKSHEET 19

Lagrange’s Theorem.

1. \( \mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\} \). Let \( H = \{0, 3\} \). Show that \( H \) is a subgroup of \( \mathbb{Z}_6 \).

2. Write out the cosets \( k + H \) for \( k \in \mathbb{Z}_6 \).

3. How many distinct cosets are there?

4. Do you notice anything interesting about the number of elements in each coset?

   For a finite group \( G \) and subgroup \( H \), the index of \( H \) in \( G \), denoted \([G : H]\), is the number of distinct cosets of \( H \) in \( G \).

5. Find \([\mathbb{Z}_6 : H]\).

   **Lagrange’s Theorem** says: for a finite group \( G \) and subgroup \( H \), \([G : H] = \frac{|G|}{|H|}\).

6. Verify Lagrange’s Theorem in the case above, for finite group \( \mathbb{Z}_6 \) and subgroup \( H \).