Factor Groups.

Define the coset $k + 5\mathbb{Z}$ of $5\mathbb{Z}$ as $\{k + l : l \in 5\mathbb{Z}\}$.

Define $\mathbb{Z}/5\mathbb{Z}$ to be the set of cosets of $5\mathbb{Z}$ in $\mathbb{Z}$. Formally, this is:

$$\mathbb{Z}/5\mathbb{Z} = \{k + 5\mathbb{Z} : k \in \mathbb{Z}\}.$$ 

1. How many distinct cosets are in $\mathbb{Z}/5\mathbb{Z}$?

We can define an “addition” for $\mathbb{Z}/5\mathbb{Z}$ in a natural way: for $k + 5\mathbb{Z}, l + 5\mathbb{Z} \in \mathbb{Z}/5\mathbb{Z}$:

$$(k + 5\mathbb{Z}) + (l + 5\mathbb{Z}) = (k + l) + 5\mathbb{Z}.$$ 

2. Show that this addition is associative.

3. Is there an “additive identity” in $\mathbb{Z}/5\mathbb{Z}$? Explain.

4. Are there “additive inverses” in $\mathbb{Z}/5\mathbb{Z}$? Explain.