Last name _____

First name _____

LARSON—MATH 401—CLASSROOM WORKSHEET 11
Division Algorithm, GCD Theorem, and Euclid's Lemma.

The **Division Algorithm** says: if $a \in \mathbb{Z}, b \in \mathbb{N}$ then there are unique integers q and r with $a = bq + r$, and $0 \leq r < b$.

q is the *quotient* and r is the *remainder*.

The *greatest common divisor* (gcd) of integers a and b (not both 0) is the largest natural number d such that d divides both a and b.

We write: $\gcd(a, b) = d$. By definition $\gcd(a, b) \geq 1$. If $\gcd(a, b) = 1$ we say that a and b are *relatively prime*.

The **GCD Theorem** says, given an integer a and positive integer b,

$$\gcd(a, b) = \min\{ak + bl : k, l \in \mathbb{Z}, ak + bl > 0\}$$

Euclid's Lemma says that if p is a prime, and a, b are integers and $p|ab$ then either $p|a$ or $p|b$.

1. Find $\gcd(5, 6)$.

2. Find integers k, l such that $\gcd(5, 6) = 5k + 6l$.

3. Find $\gcd(10, 12)$.

4. Find integers k, l such that $\gcd(10, 12) = 10k + 12l$.