Division Algorithm.

Given positive integers \(a\) and \(b\) you can write one as a multiple of the other with a non-negative remainder that’s less than what you’re “dividing” by.

Formally, the Division Algorithm says: if \(a \in \mathbb{Z}, b \in \mathbb{N}\) then there are unique integers \(q\) and \(r\) with \(a = bq + r\), and \(0 \leq r < b\).

\(q\) is the quotient and \(r\) is the remainder.

1. Suppose there is a 2nd pair of integers \(r', q'\) with \(a = bq' + r'\), and \(0 \leq r' < b\). Show \(r = r'\) and \(q = q'\).

The greatest common divisor (gcd) of integers \(a\) and \(b\) (not both 0) is the largest natural number \(d\) such that \(d\) divides both \(a\) and \(b\).

We write: \(\gcd(a, b) = d\). By definition \(\gcd(a, b) \geq 1\). If \(\gcd(a, b) = 1\) we say that \(a\) and \(b\) are relatively prime.

2. Find \(\gcd(30, 12)\).

3. Find \(\gcd(30, 31)\).