A relation $R$ on a set $X$ is a subset of $X \times X$. If $(a, b) \in R$ we sometimes write $a \sim b$, and say $\sim$ is a relation on $X$ (or even $\sim_R$ to be unambiguous).

An equivalence relation on a set $X$ is a relation with 3 properties:

1. $a \sim a$ for every $a \in X$ (reflexive).
2. If $a \sim b$ then $b \sim a$, for every $a, b \in X$ (symmetric).
3. If $a \sim b$ and $b \sim c$ then $a \sim c$, for every $a, b, c \in X$ (transitive).

If $\sim$ is an equivalence relation on $X$, we define the equivalence class of $a \in X$ as:

$$[a] = \{b \in X : a \sim b\}$$

For integers $a, b \in \mathbb{Z}$, define $a$ divides $b$ (or $b$ is a multiple of $a$) if there is an integer $n$ such that $an = b$. We write $a|b$.

We define integers $a, b \in \mathbb{Z}$ to be congruent mod $n$ ($n \in \mathbb{N}$) if $n|(a - b)$. We write:

$$a \equiv b \mod n.$$ 

1. Is it true that $5 \equiv 25 \mod 4$? Explain.

2. Show that congruence mod $n$ defines an equivalence relation on $\mathbb{Z}$.

   Let $a \sim b$ if $a \equiv b \mod n$.

   (a) Show $\sim$ is reflexive.
(b) Show $\sim$ is symmetric.

(c) Show $\sim$ is transitive.