Last name _____

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LARSON—MATH 401—CLASSROOM WORKSHEET 06
Equivalence Relations.

A *relation* R on a set X is a subset of $X \times X$. If $(a, b) \in R$ we sometimes write $a \sim b$, and say $\sim$ is a relation on X (or even $\sim_R$ to be unambiguous).

An *equivalence relation* on a set X is a relation with 3 properties:

1. $a \sim a$ for every $a \in X$ (*reflexive*).
2. If $a \sim b$ then $b \sim a$, for every $a, b \in X$ (*symmetric*).
3. If $a \sim b$ and $b \sim c$ then $a \sim c$, for every $a, b, c \in X$ (*transitive*).

If $\sim$ is an equivalence relation on X, we define the *equivalence class* of $a \in X$ as:

$$[a] = \{b \in X : a \sim b\}$$

For integers $a, b \in \mathbb{Z}$, define a *divides* b (or b is a *multiple* of a) if there is an integer n such that $an = b$. We write $a|b$.

We define integers $a, b \in \mathbb{Z}$ to be *congruent mod* n ($n \in \mathbb{N}$) if $n|(a - b)$. We write:

$$a \equiv b \mod n.$$

1. Is it true that $5 \equiv 25 \mod 4$? Explain.

2. Show that *congruence mod* n defines an equivalence relation on $\mathbb{Z}$.
   Let $a \sim b$ if $a \equiv b \mod n$.

   (a) Show $\sim$ is reflexive.

(b) Show $\sim$ is symmetric.

(c) Show $\sim$ is transitive.