Last name _____

First name _____

LARSON—MATH 391—HOMEWORK WORKSHEET 10
Test 3 Review.

You should know the following definitions, theorems, algorithms, and proofs for the test. Write out careful definitions, theorem statements, algorithms, proofs, and solutions. Turn these in at test time.

Concepts & Notation. Give the definition and an example in each case.

1. Friendship Property.
2. independent set.
3. independence number α.
4. an orthonormal representation of a graph.

Graphs

5. Draw an example of a graph with the Friendship Property. Explain.

Theorems. State the following theorems (no need to state proofs of these).

6. Friendship Theorem.
7. Interlacing Theorem.
8. Cvetkovic's Theorem.

Problems

9. $e^{i\theta} = \cos\theta + i\sin\theta$. $\omega_j = e^{\frac{i2\pi j}{4}}$. Find ω for $j = 1, 2, 3, 4$.
10. Find and simplify $\omega_j + \omega_j^{-1}$ for $j = 1, 2, 3, 4$.
11. Find $\hat{x}_j = \begin{pmatrix} \omega_j \\ \omega_j^2 \\ \omega_j^3 \\ 1 \end{pmatrix}$ for $i = 3$.

12. Find a formula for the independence number of a complete graph. Explain.

13. Find a formula for the independence number of the complete bipartite graph $K_{m,n}$. Explain.

14. Find a formula for the independence number for a cycle graph C_n. Explain.

15. Show that the Petersen graph has independence number at least 4.

16. Show that the Petersen graph has independence number no more than 4.

17. Apply Cvetkovic's Theorem to find an upper bound for the independence number of P_3.

18. Apply Cvetkovic's Theorem to find an upper bound for the independence number of C_4.

19. Prove **Lemma 2** from class: If A is a symmetric $n \times n$ matrix with eigenvalues $\lambda_1 \geq \ldots \geq \lambda_n$, and corresponding orthogonal unit eigenvectors $\{\hat{x}_i\}$, and $1 \leq p < q \leq n$ then for any unit vector $\hat{x} \in Span(\{\hat{x}_p, \ldots, \hat{x}_q\})$ we have $\lambda_q \leq \hat{x}^T A \hat{x} \leq \lambda_p$.

20. Let U, W be 2-dimensional subspaces of $\mathbb{R}^3$. Argue that there is a non-zero vector in $U \cap W$.

21. Find an orthonormal representation of C_5.