Last name _____

First name _____

LARSON—MATH 391—HOMEWORK WORKSHEET 8
Cycle Graphs.

$$e^{i\theta} = \cos\theta + i\sin\theta$$

1. Let $\theta_j = \frac{2\pi j}{4}$. Find the degree measure of θ_j for $j = 1, 2, 3, 4$.
2. Let $\omega_j = e^{\frac{i2\pi j}{4}}$. Use the formula above to find ω_j for $j = 1, 2, 3, 4$.
3. Find ω_j^2, ω_j^3 and ω_j^4 for $j = 1, 2, 3, 4$.
4. Explain why $\omega_j^{-1} = \omega_j^3$ for any choice of j.
5. Find $\omega_j + \omega_j^{-1}$ for $j = 1, 2, 3, 4$. (Hint: these are all **real** numbers).
6. Draw a cycle graph C_4 with vertices $1, 2, 3, 4$ and corresponding adjacency matrix A.
7. For each $j = 1, 2, 3, 4$ show that $\hat{x}_j = \begin{pmatrix} \omega_j \\ \omega_j^2 \\ \omega_j^3 \\ 1 \end{pmatrix}$ is an eigenvector for C_4 with corresponding eigenvalue $\omega_j + \omega_j^{-1}$.