

Last name \_\_\_\_\_

First name \_\_\_\_\_

**LARSON—MATH 391—HOMEWORK WORKSHEET 05**  
**Test 1 Review.**

You should know the following definitions, theorems, algorithms, and proofs for the test. Write out careful definitions, theorem statements, algorithms, proofs, and solutions. Turn these in at test time.

**Concepts & Notation.** Give an example in each case.

1. adjacency matrix of a graph.
2. symmetric matrix.
3. identity matrix.
4. eigenvalue  $\lambda$  and corresponding eigenvector  $\vec{x}$  of a square  $(n \times n)$  matrix  $A$ .
5. The determinant ( $\det$ ) of a square matrix.
6. The trace of a matrix.
7.  $[n]$ .
8. A permutation on  $[n]$ .
9. A transposition.
10.  $S_n$ .
11. orthogonal vectors.
12. An orthogonal matrix.
13. A normalized vector  $\vec{u}$ .
14. Regular graph.
15. The characteristic polynomial  $p(\lambda)$  of a square matrix  $A$ .
16. The degree of a vertex, the average degree, the maximum degree  $\Delta$ .
17. The Rayleigh quotient of a matrix  $A$  and vector  $\vec{x}$ . Give an example.

## Graphs

18. Draw the following graphs:  $k_2$ ,  $k_3$ ,  $k_4$ ,  $c_4$ ,  $c_5$ ,  $p_3$ ,  $2k_2$ , Petersen.

## Theorems & Proofs

19. Real Eigenvalues of a Symmetric Matrix.  
20. The Min—Max (Rayleigh-Ritz) Theorem.  
21. Trace—Eigenvalues Formula.  
22. Eigenvectors of Distinct Eigenvalues of a Symmetric Matrix Theorem.  
23. State and prove: 1<sup>st</sup> Theorem of Graph Theory.

## Problems

24. Give an example of a pair of orthonormal vectors.  
25. What nice property does a set  $\{\vec{x}_i\}$  of orthonormal vectors have?  
26. Normalize  $\vec{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ .  
27. Find the adjacency matrix  $A$  of  $p_3$ , and  $A^2$ .  
28. Find the eigenvalues of  $k_2$ .  
29.  $\sqrt{2}$  is an eigenvalue of  $p_3$ . Show how to find a corresponding eigenvector.  
30. List all permutations of  $[3]$ . Use both “arrow” and shorthand notation.  
31. Factor all permutations  $\sigma$  of  $[3]$  into transpositions and find  $sgn(\sigma)$ .  
32. The permutation  $(1234)$  can be factored as  $(12)(23)(34)$ . Make a machine diagram to show that these represent the same function.  
33. Find the characteristic polynomial of  $p_3$ .  
34. Use the determinant formula to find the determinant of the identity matrix  $I$ . There is only one non-zero term. Explain. What permutation  $\sigma$  corresponds to this term. What is the  $sgn(\sigma)$ ?  
35. The eigenvalues of a symmetric matrix  $A$  are 1, 2, 3. What are the eigenvalues of  $A^2$ ?  
36. Show that if  $\lambda$  is an eigenvalue of  $A$  with corresponding eigenvector  $\vec{x}$  then the value of the Raleigh quotient is  $\lambda$ .