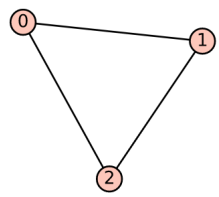
Last name _____

First name _____

LARSON—MATH 391—HOMEWORK WORKSHEET 04
Rayleigh-Ritz.

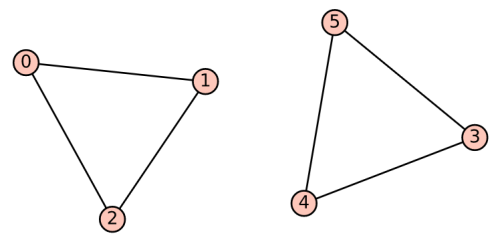


1. Find the adjacency matrix A of this graph.
2. Find the trace of A.
3. Find the characteristic polynomial $p(\lambda) = \det(A - \lambda I)$ of A using the Laplace expansion.
4. Find the characteristic polynomial $p(\lambda)$ using the determinant formula.
   (a) Find the non-zero products in this sum. (Here there's no zeros! So its just as easy or easier to list the 6 permutations and *then* find the products.)
   (b) Identify the permutation σ corresponding to each product.
   (c) Find the $sgn(\sigma)$ of each permutation.
5. Find the eigenvalues $\lambda_1, \lambda_2, \lambda_3$ of A (where $\lambda_1 \geq \lambda_2 \geq \lambda_3$).
6. Find an eigenvector $\vec{x_i}$ corresponding to each λ_i.
7. Check that the eigenvectors corresponding to different eigenvalues are orthogonal.
8. Check that the set of 3 vectors are linearly independent (do you remember how?!)
9. Write $\begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$ as a linear combination of your set of eigenvectors $\{x_i\}$.

10. Explain why these eigenvectors are a basis for $\mathbb{R}^3$.

11. Normalize your eigenvectors (find unit vectors that point in the same directions are your eigenvectors).

12. Two of your eigenvalues are the same. The eigenvectors you found probably aren't orthogonal. Can you find eigenvectors corresponding to these eigenvalues that *are* orthogonal?



Let G be this (disconnected) graph with order 6.

13. Find an adjacency matrix A of G.

14. Find the eigenvalues of A—without doing any new computations. (And definitely do not find a determinant!)

15. Find corresponding eigenvectors for your eigenvalues (they will have 6 components—but you can easily build them from your previous work).

16. For each eigenvalue-eigenvector pair, λ_i, $\vec{x_i}$, check that $A\vec{x_i} = \lambda_i \vec{x_i}$.