An *independent set* in a graph is a set of vertices which are pair-wise non-adjacent (that is, a set of vertices with no edge between any pair).

The *independence number* \( \alpha \) of a graph is the cardinality of a maximum independent set.

Cvetkovic’s Theorem says: the independence number \( \alpha \) of a graph is no more than the number of non-negative eigenvalues of the graph (and also no more than the number of non-positive eigenvalues).

1. Find a maximum independent set and \( \alpha \).

2. Find the number of non-negative eigenvalues of the Petersen Graph. Test Cvetkovic’s Theorem.

3. Find the number of non-positive eigenvalues of \( C_4 \). Test Cvetkovic’s Theorem.
4. Find a maximum independent set and $\alpha$.

5. Find the number of non-negative eigenvalues of $P_3$. Test Cvetkovic’s Theorem.

6. Find the number of non-positive eigenvalues of $P_3$. Test Cvetkovic’s Theorem.

We found that the spectrum (set of eigenvalues) of the Petersen graph was: $3^1, 1^5, -2^4$ and that $\alpha = 4$.

7. Find the number of non-negative eigenvalues of the Petersen Graph. Test Cvetkovic’s Theorem.

8. Find the number of non-positive eigenvalues of the Petersen graph. Test Cvetkovic’s Theorem.