The determinant of a square triangular matrix is the product of its diagonal entries.

Determinant Computation Rules

• The determinant of a square matrix $A$ equals the determinant of any matrix formed by a pivot operation.

• The determinant of a square matrix $A$ equals negative the determinant of any matrix formed by switching 2 rows.

Let $A = \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}$.

If there is a non-zero vector $\vec{x}$ and scalar $\lambda$ with $A\vec{x} = \lambda\vec{x}$ then $\lambda$ is an eigenvalue of $A$ and $\vec{x}$ is a corresponding eigenvector.

If $A\vec{x} = \lambda\vec{x}$, then $A\vec{x} - \lambda\vec{x} = 0$, and $(A - \lambda I)\vec{x} = 0$.

Since $\vec{x}$ is non-zero that means that $(A - \lambda I)$ is not invertible, that the RREF has a 0-row, and that $\det(A) = 0$.

1. Find $(A - \lambda I)$.

2. Use the $2 \times 2$ determinant formula to find $\det(A - \lambda I)$.
   ($\lambda$ is a variable—so your answer will have $\lambda$s in it).
3. Solve \( \det(A - \lambda I) = 0 \).

4. For each solution \( \lambda \), write the equation \( (A - \lambda I)\vec{x} = 0 \), and solve for \( \vec{x} \).

5. Check that your eigenvalue-eigenvector pairs work!