Claim: A graph is a forest if and only if every edge is a cut edge.

There are 2 claims here:

1. If a graph is a forest then every edge is a cut edge.
2. If every edge of a graph is a cut edge then the graph is a forest.

We'll need some definitions and previous results:
Def. A forest is an acyclic graph.
(*) The components of a forest are trees.

(**) A connected graph is a tree if and only if every edge is a cut edge.

Claim 1) Let $G$ be a forest. The components of $G$ are trees. Every edge of a tree is a cut edge. So every edge of $G$ is a cut edge.

Claim 2) Let $G$ be a graph. Assume every edge of $G$ is a cut edge. Let $H$ be any component of $G$. By assumption, every edge of $H$ is a cut edge. So $H$
is a tree. So it is acyclic. So $G$ is acyclic, so $G$ is a forest.