1. Log in to your Sage Cloud account.
   (a) Start the Chrome browser.
   (b) Go to http://cloud.sagemath.com and sign in.
   (c) You should see an existing Project for our class. Click on that.
   (d) Click “New”, call it h06, then click “Sage Worksheet”.

2. Monty Hall Problem Strategy #3. Write code to investigate the empirical (experimen-
   tal) probability of success for the following strategy: always pick door 0 first. If
   Monty opens door 1 switch. If he opens door 2, stick with door 1.

3. What is the (average, expected) longest streak of heads or tails if you flip a fair coin
   200 times?

4. Define a function collatz_iterates(n) which calculates the number of iterations it
   takes the Collatz function to converge to 1. Find the number of iterations it takes to
   converge for the first several powers of 3.

5. Define a function mega_prime(n) which outputs 1+the product of the first n primes.
   Find the smallest mega prime which is not itself prime. Expert: find the number of
digits of the largest prime mega prime with \( n \leq 100 \). Mega expert: conjecture the
   distribution of the mega primes.

6. How many students \( n \) are needed so that you would expect a better-than-even chance
   that more than one pair of students have the same birthday (so either there are 3 or
   more students with the same birthday, or at least 2 different pairs of students which
   share birthdays).

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7. Find 635 \( \cdot \) 629.

8. What command would you write to test whether a number \( x \) equals 0?

9. What command would you write to find the remainder of dividing an integer \( x \) by 2?

10. What command would you write to find \( \log_{10} 47 \)?

11. What command would you write to solve \( x^2 - x = 25 \).
12. What command would you write to solve the system. \[
\begin{align*}
2x + y &= 5 \\
x + 3y &= 7
\end{align*}
\]

13. Find a numerical approximation for \[
\int_2^3 t^2 e^t \, dt.
\]

14. What command would you write to find \[
\lim_{x \to 0} \frac{\sin x}{x}.
\]

15. What command would you write to find the row-reduced echelon form of the matrix \[
A = \begin{bmatrix}
2 & 1 & 5 \\
1 & 3 & 7
\end{bmatrix}.
\]

16. What command gives you the entry of matrix \(A\) in the 2nd row and 3rd column?

17. What command would you write to define a list \(L2\) which contains the integers from 2 to 55 followed by the integers from 100 to 123.

18. What command would you write to define a list \(L3\) which contains 50 zeros.

19. Define a function \(\text{square\_list}(L)\) which inputs a list \(L\) of numbers and returns a list of the squares of those numbers.

20. Define a function \(\text{three\_mult}(n)\) which tests if an integer \(n\) is a multiple of three, returns True if it is and False if it is not.

21. Define a function \(\text{list\_evens}(n)\) that returns a list of all the even numbers up to \(n\).

22. Define a function \(\text{count\_evens}(L)\) that inputs a list \(L\) of integers and counts how many of them are even.

23. Define a function \(\text{print\_numbers}(n)\) that prints , “2 is a prime”, “3 is a prime”, for each prime less than \(n\). Use a \text{while} statement.

24. What is a \text{recursive function}?

25. Define a recursive function \(\text{test\_rec}(n)\) with \(\text{test\_rec}(1)=5\) and \(\text{test\_rec}(n)=\text{test\_rec}(n-1)+17\) if \(n > 1\). Find \(\text{test\_rec}(10)\).

26. The Fibonacci sequence \(F_n\) is defined as follows \(F_0 = 0, F_1 = 1\) and \(F_n = F_{n-1} + F_{n-2}\) for \(n > 1\). Define an iterative (non-recursive) function \(\text{fib}(n)\) that computes \(F_n\) for a given input integer \(n\).

27. If \(L\) is a list of integers, what command would you give to get a scatter plot that visualizes this data? What you write should work for any list \(L\), but test it with \(L=[2,3,5,7,11]\).

28. Define a function \(\text{randlist}(n)\) which returns a list of \(n\) random numbers in \([0,1]\) sorted from smallest to largest.