

LARSON—MATH 353—CLASSROOM WORKSHEET 35
Conjecturing Invariant Bounds

1. Log in to CoCalc.
 - (a) Start the Chrome browser.
 - (b) Go to `https://cocalc.com`
 - (c) Login (**your VCU email address** is probably your username).
 - (d) You should see an existing Project for our class. Click on that.
 - (e) Click “New”, then “Worksheets”, then call it **c35**.

We can use the CONJECTURING program to conjecture upper and lower bounds for an *invariant* of a mathematical object (number, matrix, graph, etc). An *invariant* in this context means any number associated with that object. So, for instance, the determinant of a matrix is a matrix-invariant.

Inequalities show up everywhere in mathematics; famous ones include the Cauchy-Schwartz inequality. Investigating bounds can be of enormous practical importance: bounds are useful when we want to reduce a *search space* where the answer to some question may be (for instance optimizing a discrete function).

2. Load “conjecturing.py”

Here was our example from last class. This is here just as an example that you can imitate. Remember that a you can perform a *lower bound* investigation by flipping the `upperBound` switch to `False`. Any time you run conjectures you should ask: Are these conjectures true? If not, can you find a counterexample?

```
objects = [2,3,4]
invariants = [Integer.nbits, Integer.ndigits, Integer.sqrt]
invariant_of_interest = invariants.index(Integer.nbits)
conjecture(objects, invariants, invariant_of_interest, upperBound = True)
```

3. There is an updated `perfect_numbers.sage` file in the class Handouts folder. Copy this file to your Home/Root directory, load this file, and then play/experiment with these invariants. See if you can understand what each one does.
4. Now imitate our previous CONJECTURING experiment to see if you can get any interesting conjectures that might be relevant to investigating perfect numbers.

5. **Getting your classwork recorded**

When you are done, before you leave class...

- (a) Click the “Make pdf” (Adobe symbol) icon and make a pdf of this worksheet. (If Cocalc hangs, click the printer icon, then “Open”, then print or make a pdf using your browser).
- (b) Send me an email with an informative header like “Math 353—c35 worksheet attached” (so that it will be properly recorded).
- (c) Remember to attach today’s classroom worksheet!