LARSON—MATH 353–CLASSROOM WORKSHEET 31
Variance & Standard Deviation

1. Log in to CoCalc.

   (a) Start the Chrome browser.

   (b) Go to `https://cocalc.com`

   (c) Login (**your VCU email address** is probably your username).

   (d) You should see an existing Project for our class. Click on that.

   (e) Click "New", then "Worksheets", then call it **c31**.

Variance & Standard Deviation

Suppose you flip a coin 100 times. You'd expect 50 heads. You'd also expect to rarely get exactly 50 heads. What is the *average* difference from 50? What can you say about the *distribution* of differences?

Let's define some functions so we can *experiment*.

2. Define a function `coin_flips(n)` that produced n "coin flips" (at this point we just use randomly produced 0s and 1s and interpret 1s as "heads".

3. Define a function `heads(flips)` that takes a sequence of coin flips and report how many heads there are.

4. We'll want to repeat our 100-coin-flip experiment many many times, so maybe it would be useful to define a function `experiments(n,m)` that repeats a n-flip experiment m-times and returns a list of the number of heads from each of these experiments. (The expected number of heads in each experiment is $\frac{n}{2}$.)

5. Test these functions to see if they are working and use `histogram` to generate some pictures.

6. One measure of "difference from the mean" is the absolute value of the actual number of heads from the expected number. Write a function `experiment_differences(n,m)` that that repeats a n-flip experiment m-times and returns a list of this difference of the produced number of heads from each of these experiments from their expected number.

   Test this functions to see if it is working and use `histogram` to generate some pictures.

7. Another measure of the "difference from the mean" is the *standard deviation*. First we'll define the *variance* of a sequence: its the average of the squared differences from the mean (for our flip experiments it will be the sum of the squared differences from the expected number of heads).

   Since these are "squared quantities" we take the square-root in order to get a number of the right "dimension". This square-root-of-the-variance is the standard deviation σ.

   Write a function that produces the standard deviation of a sequence of coin tosses. And test it.

   The formula for the normal distribution with mean μ (expected number of heads) and standard deviation σ is:

$$f(x) = \frac{e^{\frac{-(x-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}$$

8. Plot this function for values of μ and σ from our coin tossing experiments. Can you guess how this function might be interpreted?

9. What does this formula say for an experiment with mean 0 and standard deviation 1?

10. Can we "normalize" our coin flipping experiment to produce a distribution with mean 0 and standard deviation 1?

11. How can we compare our actual experimental results with what we get from this formula. What does it *mean*?

12. Can we use experiments and this formula to find another estimate of π (and why does π show up in this formula)???

13. **Getting your classwork recorded**

    When you are done, before you leave class...

    (a) Click the "Make pdf" (Adobe symbol) icon and make a pdf of this worksheet. (If Cocalc hangs, click the printer icon, then "Open", then print or make a pdf using your browser).

    (b) Send me an email with an informative header like "Math 353—c31 worksheet attached" (so that it will be properly recorded).

    (c) Remember to attach today's classroom worksheet!