LARSON—MATH 353–CLASSROOM WORKSHEET 28
Digital Dice & Optimal Stopping

1. Log in to CoCalc.
   (a) Start the Chrome browser.
   (b) Go to `https://cocalc.com`
   (c) Login (**your VCU email address** is probably your username).
   (d) You should see an existing Project for our class. Click on that.
   (e) Click "New", then "Worksheets", then call it **c28**.

Nahin's Digital Dice

2. **The Curious Coin-Tossing Game**. There are three players each with a stack of coins. They each flip one. If they all get Heads or all get Tails they play again. Otherwise the "odd man out" (the one who got say Heads while the other two get Tails) get the other's two coins (so her pile goes up two, while their piles of coins each go down one). If they start with k, l and m coins, how many iterations of this game does it take on average for (at least) one player to lose all of her coins?

   We showed mathematically that the number of iterations in the case $k = l = m = 1$ is at least $\frac{85}{64} \sim 1.328$. We then confirmed this by repeating a million experiments.

3. What about if $k = l = m = 2$, $k = l = m = 3$ and $k = l = m = 10$?

4. We've assumed that this was a *fair* coin. How can we modify our code in order to have our players play with coins where the probability of heads is p (maybe $p = 0.4$)?

5. **Optimal Stopping**. A coin is flipped n times. Let s_n be the number of Heads that come up. At any point you can stop and collect $\frac{s_n}{n}$ dollars (so the max would be \$1). If you get a Heads on the first flip you should stop. If you get Tails on the first flip, you should flip again.

   What *strategy* should you use to decide whether it is optimal to stop (whether you would win the *expected* maximum if you stopped at that point).

6. Zeilberger notes that the game must stop after some finite number N of flips. Write code that simulates the average maximum value of $\frac{s_n}{n}$ given that k heads and m tails have already been flipped (so there are $N-k-m$ flips remaining and we know $\frac{s_n}{n} \geq \frac{k}{k+m}$ We also know the expected average maximum value when $N=1$ is $\frac{1}{2}$).

7. How can we use information that we get from our simulations to design an optimal stopping *strategy*.

8. **Getting your classwork recorded**

   When you are done, before you leave class...

   (a) Click the "Make pdf" (Adobe symbol) icon and make a pdf of this worksheet. (If Cocalc hangs, click the printer icon, then "Open", then print or make a pdf using your browser).

   (b) Send me an email with an informative header like "Math 353—c28 worksheet attached" (so that it will be properly recorded).

   (c) Remember to attach today's classroom worksheet!