1. Log in to your Sage Cloud account.
   
   (a) Start the Chrome browser.
   (b) Go to http://cloud.sagemath.com and sign in.
   (c) You should see an existing Project for our class. Click on that.
   (d) Click “New”, call it c28, then click “Sage Worksheet”.

Recall that the degree of a vertex is the number of edges incident to the vertex. The minimum degree $\delta$ and maximum degree $\Delta$ are, respectively, the largest and smallest of these.

Now we’ll add a theory parameter in order to tell CONJECTURING about mathematical facts that we’ve proved—and that we want it to improve on.

2. When we ran the following script we got the conjecture:

   $$\text{size}(x) \leq \frac{1}{2} \cdot \text{min\_degree}(x) \cdot \text{order}(x)$$

   This conjecture is true (as we’re interpreting it—for connected graphs). Now we’ll tell the program that we know the conjecture is true.

   ```python
   load('conjecturing.py')
   k3=graphs.CompleteGraph(3)
   k4=graphs.CompleteGraph(4)
   k5=graphs.CompleteGraph(5)

   def min_degree(g):
       return min(g.degree())
   def max_degree(g):
       return max(g.degree())

   k3=graphs.CompleteGraph(3)
   k4=graphs.CompleteGraph(4)
   k5=graphs.CompleteGraph(5)

   half_max_deg_times_order = lambda g: 1/2*max_degree(g)*order(g)
   theory=[half_max_deg_times_order]
   invariants = [Graph.size, Graph.order, min_degree, max_degree]
   objects = [k3,k4,k5]
   conjecture(objects, invariants, invariants.index(Graph.size), theory=theory)
   ```
3. Choose a conjecture that interests you and write it down.

4. Draw lots of examples of small graphs to get a feeling for the truth of the conjecture and, possibly, what a counterexample would look like.

5. Is true for all connected graphs? Why? If not, try to find a counterexample. If you think its true, try to prove it!

To get *lower bound* conjectures for an invariant we added the *parameter* `upperBound=False` to the call of the `conjecturing` function. We can also add a *theory* parameter to the lower bound.

It's true that, for all connected graphs that: \( \text{size}(x) \geq \text{max\_degree}(x) \)

6. Add this fact to the program and generate lower bound conjectures for the size of a connected graph. (How did you do it?)

7. Choose a conjecture that interests you and write it down.

8. Is true for all connected graphs? Why? If not, try to find a counterexample. If you think its true, try to prove it!