1. Log in to your Sage Cloud account.
   (a) Start the Chrome browser.
   (b) Go to http://cloud.sagemath.com and sign in.
   (c) You should see an existing Project for our class. Click on that.
   (d) Click “New”, call it c27, then click “Sage Worksheet”.

We’ll add more invariants in order to get more interesting—and more finely-tuned—conjectures. Let’s add minimum and maximum degree. Recall that the degree of a vertex is the number of edges incident to the vertex. The minimum degree $\delta$ and maximum degree $\Delta$ are, respectively, the largest and smallest of these.

2. Generate upper-bound conjectures for (graph) size in terms of (graph) order, minimum and maximum degree:
   
   ```python
   load('conjecturing.py')
   k3=graphs.CompleteGraph(3)
   k4=graphs.CompleteGraph(4)
   k5=graphs.CompleteGraph(5)
   def min_degree(g):
       return min(g.degree())
   def max_degree(g):
       return max(g.degree())
   k3=graphs.CompleteGraph(3)
   k4=graphs.CompleteGraph(4)
   k5=graphs.CompleteGraph(5)
   invariants = [Graph.size, Graph.order, min_degree, max_degree]
   objects = [k3,k4,k5]
   conjecture(objects, invariants, invariants.index(Graph.size))
   ```

If a conjecture is true, the only way to be certain is to prove it. If it is false, the only way to be certain of that is to find an example that demonstrates falsity (a counterexample).

3. Choose a conjecture that interests you and write it down.
4. Draw lots of examples of small graphs to get a feeling for the truth of the conjecture and, possibly, what a counterexample would look like.

5. Is true for all connected graphs? Why? If not, try to find a counterexample. If you think its true, try to prove it!

To get lower bound conjectures for an invariant we add the parameter upperBound=False to the call of the conjecturing function (its default value is True, so you automatically get upper bounds unless you explicitly ask for lower bounds).

6. Evaluate:

   conjecture(objects,invariants,invariants.index(Graph.size),upperBound=False)

7. Choose a conjecture that interests you and write it down.

8. Draw lots of examples of small graphs to get a feeling for the truth of the conjecture and, possibly, what a counterexample would look like.

9. Is true for all connected graphs? Why? If not, try to find a counterexample. If you think its true, try to prove it!