

LARSON—MATH 353—CLASSROOM WORKSHEET 23
Memoization—Digital Dice

1. Log in to CoCalc.
 - (a) Start the Chrome browser.
 - (b) Go to `https://cocalc.com`
 - (c) Login (**your VCU email address** is probably your username).
 - (d) You should see an existing Project for our class. Click on that.
 - (e) Click “New”, then “Worksheets”, then call it **c23**.

Avoiding recomputation: Dictionaries & Memoization

We can make a recursive function like our recursive Fibonacci function more efficient by storing values and then checking if they’ve already been computed before (instead of inefficiently re-computing them). A great place to store values like this is a Python *dictionary*. A dictionary consists of {KEY:VALUE} pairs, enclosed in set brackets, with pairs separated by commas.

Better even than maintaining a dictionary external to our recursive function is to make a new closely-related function with its own built-in dictionary.

2. Code our recursive fibonacci number function and check that it works.

```
def fib(n):
    if n==0 or n==1:
        return n
    else:
        return fib(n-1)+fib(n-2)
```

Recall that we could not compute `fib(40)` with this function.

What we can do is use this inefficient function as input to a function that takes any function f and outputs a helper function that does what f does—but first creates a dictionary, returns previously computed values from the dictionary, and stores any not-previously-computed values to the dictionary.

3. Try the following code:

```
def memoize(f):
    memo = {}
    def helper(x):
        if x not in memo:
            memo[x] = f(x)
        return memo[x]
    return helper

fib=memoize(fib)
```

`fib` is now actually the helper function returned by the `memoize` function. Make sure that it works for small values that you know like `fib(0)`, `fib(1)`, `fib(2)`, `fib(3)`, etc.

4. Now try `fib(40)` (Shazaam!)

Nahin's Digital Dice

5. **Preface: Presidents.** The quiz question asks a student to match 24 Presidents to 24 years they started their presidency. The student randomly guesses (matching each President to one of the 24 years). How many Presidents will the student match correctly on average?

Define a function `experiment_presidents()` that simulates a random choice of 24 Presidents to 24 years and outputs the number of correct matches. Then repeat this experiment lots of times and calculate the average number of correct matches.

If you didn't previously succeed at coding this here's what I would try. Create a list `presidents = [1..24]` representing the 24 presidents. Let `years = []` be the randomly chosen years. So now we have to figure out how to choose the years, and then we have to check the number of matches, and return that (then we will repeat this one experiment lots of times).

How to make the random choices without any repeats? You could use the `random.choice()` function we used before, together with the `.remove()` list method we used before to remove a chosen President from the list of yet-to-be-chosen `presidents`.

6. **Preface: Estimating π .** How can π be calculated? Well it can be *defined* by the area formula for a circle: $A = \pi r^2$. Then π is the square root of the area of a unit circle. How can we estimate that?

We can let this circle be centered at the origin. It would be enough to find the area of the quarter-circle in the 1st quadrant. This slice is in the square $[0, 1] \times [0, 1]$. Define a function `experiment_pi(n)` that simulates a random choice of a point $(x, y) \in [0, 1] \times [0, 1]$. If $x^2 + y^2 \leq 1$ call that a "win" and output the ratio of wins to experiments. Try larger and larger values of n . Make a scatter plot of this behavior.

7. **The Clumsy Dishwasher.** There are 5 dishwashers. 5 dishes are broken one week—4 by the same dishwasher! Is the dishwasher clumsy or unlucky? What is the probability a dishwasher will break at least 4 dishes—under the assumption that the dishes are broken randomly (that is, with equal probability)?

Define a function `experiment_dishwasher()` that simulates this situation. What should your function output? Repeat this experiment lots of times. What is the answer to our question?

8. **Getting your classwork recorded**

When you are done, before you leave class...

- Click the "Make pdf" (Adobe symbol) icon and make a pdf of this worksheet. (If Cocalc hangs, click the printer icon, then "Open", then print or make a pdf using your browser).
- Send me an email with an informative header like "Math 353—c23 worksheet attached" (so that it will be properly recorded).
- Remember to attach today's classroom worksheet!