LARSON—MATH 353–CLASSROOM WORKSHEET 21
Perfect Numbers—Conjecturing—Ore.

1. Log in to CoCalc.

   (a) Start the Chrome browser.

   (b) Go to `https://cocalc.com`

   (c) Login (**your VCU email address** is probably your username).

   (d) You should see an existing Project for our class. Click on that.

   (e) Click "New", then "Worksheets", then call it **c21**.

Perfect Numbers Investigation

A number (integer) n is *perfect* if the sum of its proper divisors (the divisors less than n) equals n (or equivalently that the sum of all divisors equals $2n$). 6 is the smallest perfect number.

What can we do to advance our perfect numbers investigation?

2. Let's start by making a sketch of the numbers that we have been using as data, and their values, factors, and properties. This might usefully inspire new ideas. Let's also add the `abundance_index` from Prof. Nielsen's talk.

   We've been investigating *sufficient conditions* for a number to be perfect.

   (a) We can add more Properties. (How?)

   (b) We can generate *necessary* condition conjectures.

   (c) We can investigate the non-perfect numbers. (This might make some sense as the perfect numbers are very rare).

   (d) We can add more Integers/Objects. Which ones? We can't add them *all*. Every one we add will add to the computation time for each run. We should add them with some plan, for some systematic reasons.

   As we go we are storing all tested definitions and properties in `perfect_numbers.sage`. Several others have been added there, as well as the command `load("conjecturing.py")`—so that program will be loaded every time `perfect_numbers.sage` is loaded.

3. Start-up. `load("perfect_numbers.sage")` to load what we have so far (my current copy is in the Handouts folder. Your copy may have different functions. It should be in your Root/Home directory.

4. What other integer properties do we already know—or that we can cook up—to add to our Properties list (and, after suitable testing, to `perfect_numbers.sage`)? See the new *invariants* added there—these might be useful for inventing useful new properties.

5. **Necessary Conditions**. We can also investigate *necessary* conditions for a number to be perfect. Try:

```
#Necessary Condition Conjectures. 1st Run.

Integers = [6, 28, 496, 8128, 8, 15, 7, 25, 12, 140]

Properties = [is_perfect, Integer.is_prime, Integer.is_square, Integer.is_squ

Prop_of_interest = Properties.index(is_perfect)

propertyBasedConjecture(Integers,Properties,Prop_of_interest,sufficient=False
```

   If a conjecture is true, the only way to be certain is to *prove* it. If it is false, the only way to be certain of that is to find an example that demonstrates falsity (a *counterexample*).

6. What conjectures do you get? (Are they true? If not find a counterexample and add it to Sage. Then re-run to get new conjectures.)

7. Try **your own** investigation. Do several runs, find and add counterexamples. Try and **prove** conjectures that you can't find counterexamples to.

8. **Getting your classwork recorded**

   When you are done, before you leave class...

   (a) Click the "Make pdf" (Adobe symbol) icon and make a pdf of this worksheet. (If Cocalc hangs, click the printer icon, then "Open", then print or make a pdf using your browser).

   (b) Send me an email with an informative header like "Math 353—c21 worksheet attached" (so that it will be properly recorded).

   (c) Remember to attach today's classroom worksheet!