1. Log in to your Sage Cloud account.

(a) Start the Chrome browser.
(b) Go to http://cloud.sagemath.com and sign in.
(c) You should see an existing Project for our class. Click on that.
(d) Click “New”, call it c20, then click “Sage Worksheet”.

**Question 1:** What’s the probability that a graph is connected? We’ve seen that the order (number of vertices) is essential to the answer of this question. We found that the percentage of connected graphs with order 100 is greater than the percentage of graphs with order 3.

**Question 2:** What’s the probability that a graph with order \( n \) is connected?

The answer is:

\[
\frac{\text{The number of connected graphs of order } n}{\text{The number of graphs with order } n}
\]

**Idea 1:** We can estimate this proportion but putting all the order \( n \) graphs in a hat, choosing a random sample, and calculating the proportion of graphs in the sample that are connected.

**Note:** As \( n \) gets large the number of graphs with order \( n \) gets very large. Then the obvious way of calculating the number in the numerator—generate all order \( n \) graphs and check if each is connected—is a non-starter, finite, but practically impossible.

**Idea 2:** Instead of putting all the order \( n \) graphs in a hat, we can generate individual random graphs by starting with \( n \) vertices, and for each pair of vertices, flipping a coin—if the coin comes up heads, we put an edge between those vertices. So we get a choice of a graph from our hat of graphs without actually generating all of the order \( n \) graphs. Then we can check if this random graph is connected, and repeat several times.

**Question 3:** What’s the probability that a random graph with order \( n \) is connected?

We may want a graph to model some real-world situation. We talked about friendship graphs: the vertices are people, with an edge between a pair of vertices if the corresponding people are friends. Maybe in real-life, its not true that there is an even chance that a pair is friends (what you’d get with a coin flip from a fair coin). Maybe we know (empirically) that the probability \( p \) that a pair of people are friends is \( \frac{1}{3} \). We can do something similar to our random graph generation procedure, and just use a weighted coin that comes up heads \( \frac{1}{3} \) of the time. Here \( p \) is called the edge-probability.

**Question 4:** What’s the probability that a random graph with order \( n \) and edge probability \( p \) is connected?
Result 1: The probability that a random graph with order 3 and edge probability $p = \frac{1}{2}$ is connected is 0.50.

Result 2: The probability that a random graph with order 100 and edge probability $p = \frac{1}{2}$ is connected is 1.0.

2. Find the probability that a graph with order 3 and edge probability $\frac{1}{3}$ is connected. We’ll do this analytically first (because we can). List all order 3 graphs and their corresponding probabilities. Add up the probabilities of the connected ones.

3. Find the probability that a random graph with order 3 and edge probability $\frac{1}{3}$ is connected. We’ll do this experimentally. Generate some number of graphs by flipping weighted coins and keep track of the percentage that are connected.

4. Find the probability that a random graph with order 100 and edge probability $\frac{1}{3}$ is connected. We’ll do this experimentally. Generate some number of graphs by flipping weighted coins and keep track of the percentage that are connected.

5. Find the probability that a graph with order 3 and edge probability $\frac{1}{4}$ is connected. Do this analytically. List all order 3 graphs and their corresponding probabilities. Add up the probabilities of the connected ones.

6. Find the probability that a random graph with order 3 and edge probability $\frac{1}{4}$ is connected. Do this experimentally. Generate some number of graphs by flipping weighted coins and keep track of the percentage that are connected.