Last name ______________________

First name ______________________

LARSON—MATH 350–CLASSROOM WORKSHEET 05
Combinations, Permutations, Binomial Coefficients.

1. Log in to your Sage Cloud account.
   (a) Start Firefox or Chrome browser.
   (b) Go to `http://cloud.sagemath.com`
   (c) Click "Sign In".
   (d) Click project **Math 350**.
   (e) Click "New", call it **s05**, then click "Sage Worksheet".

2. Let L be the set consisting of the numbers 2,4 and 5. We will represent this set as a Sage list. Evaluate `L=[2,4,5]`. Now evaluate `L` to display the list.

3. Let's find all of the orderings or *permutations* of this list. Evaluate `Permutations(L)`. Not much happened. Let's force Sage to display the results. Evaluate `Permutations(L).list` What is the output?

4. Let's find all of the (unordered) subsets or *combinations* of this list. Evaluate `Combinations(L)`. Again, force Sage to display the results. What is the output?

5. Now let's find all of the subsets of with 2 elements. Evaluate `Combinations(L,2).list()`.

6. The integers from 1 to n can be represented using the list [1..n]. Evaluate `[1..10]`. What do you get?

7. To find all of the 3 element subsets of the integers from 1 to 10, we can evaluate `Combinations([1..10],3).list()`. Well that's a long list! What if we just want to know how many 3-subsets there are? Evaluate `Combinations([1..10],3).cardinality()` What do you get?

8. We know that the binomial coefficients $\binom{n}{k}$ are a notation for the number of k-subsets of an n-set. So our previous question is equivalent to finding $\binom{10}{3}$. We can do that with the `binomial` command. Evaluate `binomial(10,3)`.

9. Calculate the number of 10-subsets of a 20-set. What command did you use?

10. Calculate the number of 100-subsets of a 200-set. It's ginormous. Don't write it down! Write the first few digits.

11. Let's estimate it by finding out how many digits it has. First give the number a name: evaluate `b=binomial(200,100)`. Now we'll find the base-10 log of b. Evaluate `log(b,10`. What do you get?

12. Sage attempts to give (and store) precise, unrounded answers—unless you force it to give a numerical approximation. Evaluate `n(log(b,10))` to get this approximation. Round up to get the number of digits. What is that?

13. So what is a reasonable lower bound for the number of 100-subsets of a 200-set?

14. Now let's add up all of the binomial coefficients for $n = 5$. Evaluate `sum([binomial(5,k) for k in [0..5]])`. What did you get? This should agree with a theorem we proved—what does the theorem say?

15. Add up the left-side binomial coefficients for $n = 10$ (so for $k \in \{0, \ldots, 5\}$). What command will you use?