# LARSON—MATH 350—Test 3 REVIEW

On all questions you will be asked to **show your work**. You may be asked to **explain** your answers.

**Definitions, Notation**.

1. Sample Space & Rules for Probability.
2. A *convex* polygon.
3. A collection of lines in *general* position or or *standard* position.
4. $s$-clique (in a graph whose lines are either solid or dotted).
5. $d$-clique (in a graph whose lines are either solid or dotted).
6. $R(s, d)$.

**Conjectures, Theorems, Ideas**.

1. Law of Large Numbers
2. The number of intersections of $n$ points where no three are on the same line.
3. The number of regions formed by $n$ lines in general position.
4. State a conjectured formula for the fewest number of points required in order to *guarantee* that $k$ of the points form a convex $k$-gon (Happy End Conjecture)?
5. (State) the Party Problem
6. $R(s, d)$ Existence Theorem.

**Proofs**.

1. For any 5 points in the plane, there are 4 which form a convex quadrilateral (4-gon).
2. $R(2, 3) = 3$.
3. $R(3, 3) > 5$.

**Sample Problems**.

1. Consider the experiment of flipping a coin 5 times. Assume heads and tails are equally likely on each toss. Are the events of getting an odd number of heads and an even number of heads independent?
2. Show that $\emptyset$ is independent of every event.
3. Prove that if $A$ and $B$ are (mutually) exclusive events, then $P(A)+P(B) = P(A \cup B)$.
4. Simplify:

$$\frac{\binom{n+1}{4}}{\binom{n}{4}}$$

5. Simplify this expression for the ratio of consecutive binomial coefficients:

$$\frac{\binom{n}{k+1}}{\binom{n}{k}}$$

6. Show that:

$$\binom{2m}{m-t-1} \leq c\binom{2m}{m-1}, \text{ where } c = \frac{\binom{2m}{m-t}}{\binom{2m}{m}}, \text{ for some fixed } t \geq 0.$$

7. Use the Law of Large Numbers to find $n$ so that the probability that the number of heads in $n$ flips is between 49% and 51% heads is at least 0.95.

8. Draw an example of a non-convex pentagon.

9. Draw a convex 7-gon. How many points of intersection do the diagonals have (inside the figure, assuming no 3 diagonals meet in the same point)?

10. How many diagonals does a convex n-gon have?

11. Draw four lines in standard position, and count the regions formed by lines.

12. Find 6 points in the plane, where no 5 of the points form a convex pentagon (5-gon).

13. Given $n \geq 3$ lines in the plane in general position, explain why there must be at least one triangle among the regions the lines divide the plane into.

14. Find $R(2, n)$.

15. What are the two things you must do in order to show that $R(s, d) = n$ you must do 2 things