Write up nice, complete answers. Email them as a single document (preferably pdf).

**Notation** (just for reference; some from our book, and some we invented for class).

1. $C_n$ is the number of triangulations of a convex $(n+2)$-sided polygon, or $(n+2)$-gon.
2. $T_n$ is the triangular benzenoid with $n$ layers.
3. $M_n$ is the number of complete matchings in $T_{n-1}$.
4. $B_n$ is the number of bracketings with $n+1$ x’s, $n$ left parentheses, and $n$ right parentheses.
5. $D_n$ is the number of Dyck paths of length $2n$.
6. $BS_n$ is the number of ballot sequences of length $2n$.

**Definitions.** Give a careful definition and example.

1. *Dyck path.*
2. *ballot sequence.*
3. *strict ballot sequence.*
4. *graph.*
5. *planar graph.*
6. *benzenoid.*
7. *matching* (in a graph).
8. *complete matching* (in a graph).
9. *Non-Crossing Murasaki diagram.*

**Problems.** Explain your answers (that’s half of mathematics!)

10. Find all bracketings of strings of 4 x’s ($n = 3$ left parentheses, and $n = 3$ right parentheses).
11. For each bracketings of strings of 4 x’s find the associated binary tree.
12. Find all bracketings of strings of 5 x’s ($n = 4$ left parentheses, and $n = 4$ right parentheses).
13. How many different sequences of 4 1’s and 4 -1’s are there?
14. How many strict ballot sequences are there with five 1’s and four -1’s?
15. What would a *bijective proof* that $M_n = D_n$ do (what would it show)?
16. Give a bijective proof that $M_2 = D_2$.
17. Find all complete matchings in $T_2$. Explain your system (do not use the fact that $M_3 = C_3$).
18. Find all Dyck paths of length 6.
19. For each Dyck path of length 6 find the corresponding complete matching in $T_2$.
20. How many complete matchings are there of triangular benzenoids $T_n$?
21. How many complete matchings are there of $T_7$?
22. What does the formula $C_n = \frac{1}{n+1} \binom{2n}{n}$ count?
23. Find all Non-Crossing Murasaki diagrams with 2 vertical lines.
24. Find all Non-Crossing Murasaki diagrams with 4 vertical lines.