1. 7 shift

2. \(a_1 a_2 a_3 a_4 a_5 a_6 a_7\)
   \(a_2 a_3 a_4 a_5 a_6 a_7 a_1\)
   \(a_3 a_4 a_5 a_6 a_7 a_1 a_2\)
   \(a_4 a_5 a_6 a_7 a_1 a_2 a_3\)
   \(a_5 a_6 a_7 a_1 a_2 a_3 a_4\)
   \(a_6 a_7 a_1 a_2 a_3 a_4 a_5\)
   \(a_7 a_1 a_2 a_3 a_4 a_5 a_6\)

3. 7 total 1's and -1's
   4 "spots" for 1's
   3 spots for -1's

So, \(\binom{7}{4}\) or \(\binom{7}{3}\)

\[\text{Note} \quad \binom{7}{4} = \binom{7}{3} = \frac{7!}{3!4!} = 35\]

5. For any sequence of 4 1's
and 3 -1's there are 7 shifts of the sequence.

- Since each sequence corresponds to a strict ballot sequence (we proved there is a cyclic shift that results/yields/is a strict ballot sequence) this means there are 7 sequences that yield the same strict ballot sequence.

- Since there are 35 sequences total, and each sequence corresponds to 7 cyclic shifts and one strict ballot sequence, we get \[ \frac{35}{7} = 5 \] strict ballot sequences.
Now, let's find/construct the strict ballot sequences with 4 1's and 3 -1's. We argued that strict ballot sequences must start with 11 and end with -1. So the form is:

\[
\begin{array}{cccc}
11 & 1 & 1 & -1 \\
\uparrow & \uparrow & \uparrow & \uparrow \\
51 & 52 & 53 & 54
\end{array}
\]

and there are 4 "spots" (51, 52, 53, 54) to fill with two 1's and two -1's.

Case 1 51 = 1

Form 1111 -- -- --

Note: the one remaining 1 can go in any of the 3 spots—in each case all partial sums will be positive.
Get: \[
\begin{array}{cccc}
1 & 1 & 1 & -1 \\
1 & 1 & -1 & 1 \\
1 & -1 & -1 & 1 \\
1 & -1 & -1 & -1 \\
\end{array}
\]

Case 2 \[S_1 = -1\]

form: \[11 -1 \_ \_ \_ -1\]

\[\uparrow \]

Note: \[S_2\] cannot be \(-1\), as the last partial sum will be 0. So the form must be

\[11 -11 -1 -1\]

Now the remaining \(1\) can go in either spot — all partial sums will be positive.

Get:

\[
\begin{array}{cccc}
1 & 1 & 1 & 1 \\
1 & 1 & 1 & -1 \\
1 & -1 & -1 & 1 \\
1 & -1 & -1 & -1 \\
\end{array}
\]

And there are no other cases. That's the 5 strict ballot sequences.
\[ C_3 = \frac{1}{7} \cdot \left( \frac{7}{3} \right) = \frac{1}{3} \cdot 35 = 5 \]