Last name __________________
First name __________________

LARSON—MATH 350—HOMEWORK WORKSHEET 05

Write up nice, complete answers. Email them as a single document (preferably pdf).

Notation.
1. $C_n$ is the number of triangulations of a convex $(n+2)$-sided polygon, or $(n+2)$-gon.
2. $BT_n$ is the number of binary trees with $n$ nodes.
3. $PT_n$ is the number of plane trees with $n + 1$ nodes.
4. $BS_n$ is the number of ballot sequences of length $2n$.
5. $B_n$ is the number of bracketings with $n + 1$ x’s, $n$ left parentheses, and $n$ right parentheses.
6. $D_n$ is the number of Dyck paths of length $2n$.

Problems.
1. How many divisors does 1000 have? Do not find them and do not list them. Explain your answer.

2. We showed that $C_n = \frac{1}{n+1}\binom{2n}{n}$. We will investigate the equivalent expression $C_n = \frac{1}{2n+1}\binom{2n+1}{n}$. Show they are equivalent by showing that

$$\frac{1}{n+1}\binom{2n}{n} = \frac{1}{2n+1}\binom{2n+1}{n}.$$

(Use the formula we proved for binomial coefficients and algebra).

3. We showed that $PT_n \leq BT_n$ by showing how to associate a unique binary tree with $n$ nodes to each plane tree with $n + 1$ nodes. List all 14 plane trees with 5 nodes and find the associated binary tree. Show your work.

4. We claimed that this process can be reversed: that is, for every binary tree with $n$ nodes we can associate a plane tree with $n + 1$ nodes (and thus $PT_n \geq BT_n$). Pick a binary tree with 4 nodes. Find the associated plane tree with 5 nodes. Explain.

5. We showed that $B_n \leq BT_n$ by showing how to associate a unique binary tree with $n$ nodes to each bracketing with $n + 1$ x’s. List all 14 bracketings with 5 x’s and find the associated binary tree. Show your work.

6. We claimed that this process can be reversed: that is, for every binary tree with $n$ nodes we can associate a bracketing with $n + 1$ x’s (and thus $B_n \geq BT_n$). Pick a binary tree with 4 nodes. Find the associated bracketing with 5 x’s. Explain.