Last name _____

First name _____

LARSON—MATH 350—HOMEWORK WORKSHEET 05

Write up nice, complete answers. Email them as a **single** document (preferably pdf).

Notation.

1. C_n is the number of triangulations of a convex $(n+2)$-sided polygon, or $(n+2)-$gon.
2. BT_n is the number of binary trees with n nodes.
3. PT_n is the number of plane trees with $n+1$ nodes.
4. BS_n is the number of ballot sequences of length $2n$.
5. B_n is the number of bracketings with $n+1$ x's, n left parentheses, and n right parentheses.
6. D_n is the number of Dyck paths of length $2n$.

Problems.

1. How many divisors does 1000 have? Do not find them and do not list them. Explain your answer.

2. We showed that $C_n = \dfrac{1}{n+1}\dbinom{2n}{n}$. We will investigate the equivalent expression $C_n = \dfrac{1}{2n+1}\dbinom{2n+1}{n}$. Show they are equivalent by showing that
$$\frac{1}{n+1}\binom{2n}{n} = \frac{1}{2n+1}\binom{2n+1}{n}.$$
(Use the formula we proved for binomial coefficients and algebra).

3. We showed that $PT_n \leq BT_n$ by showing how to associate a unique binary tree with n nodes to each plane tree with $n+1$ nodes. List all 14 plane trees with 5 nodes and find the associated binary tree. Show your work.

4. We claimed that this process can be reversed: that is, for every binary tree with n nodes we can associate a plane tree with $n+1$ nodes (and thus $PT_n \geq BT_n$). Pick a binary tree with 4 nodes. Find the associated plane tree with 5 nodes. Explain.

5. We showed that $B_n \leq BT_n$ by showing how to associate a unique binary tree with n nodes to each bracketing with $n+1$ x's. List all 14 bracketings with 5 x's and find the associated binary tree. Show your work.

6. We claimed that this process can be reversed: that is, for every binary tree with n nodes we can associate a bracketing with $n+1$ x's (and thus $B_n \geq BT_n$). Pick a binary tree with 4 nodes. Find the associated bracketing with 5 x's. Explain.