Find $C_3$

$$C_3 = -\frac{1}{2} \left( \frac{1/2}{4} \right) (-4)^4$$

$$= -\frac{1}{2} \left( \frac{1}{2} \cdot 3 \cdot \frac{1}{2} \cdot 3 \right) \left( \frac{1}{2} \cdot 3 \right)$$

$$= \frac{(-2)(1-4)(1-6)}{2^4 \cdot 4 \cdot 3 \cdot 2}$$

$$= -\frac{3 \cdot 5}{2^7} = -\frac{5}{2^7}$$

So

$$-\frac{1}{2} \left( \frac{1/2}{4} \right) (-4)^4 = \frac{1}{2} \cdot \frac{5}{2^5} \cdot (2^2)^4$$

$$= \frac{5 \cdot 2^8}{2^8} = \boxed{5}$$
Binary Trees with 4 nodes

All have the form

Case $k = 0$ (so $3-k = 3$)

Case $k = 1$ (so $3-k = 2$)

Case $k = 2$ (so $3-k = 1$)
Case $K = 3$

$B_{n+1} = \# \text{of binary trees with } n+1 \text{ nodes}$

Show: $B_{n+1} = \sum_{k=0}^{n} B_k \cdot B_{n-k}$

- Let $T$ be a binary tree with $n+1$ nodes.
- So it has a root node and $n$ other nodes.
- $K$ of these nodes will be
in the left subtree, and 
\( k = 0, 1, \ldots, n \)

- there are \( n-k \) remaining 
  nodes for the right subtree

- there are \( BT_k \) possibilities
  for the left subtree for
  each choice of \( k \)

- then there are \( BT_{n-k} \)
  possibilities for the right
  subtree

- then for each \( k = 0, \ldots, n \)
  there are \( BT_k \cdot BT_{n-k} \)
  possibilities for a left
  subtree and a right subtree.
Summing up over all possible values for $k$ we get:

$$B_{T_n+1} = \sum_{k=0}^{n} B_T k \cdot B_{T_n-k}$$