Last name ________________________

First name ________________________

LARSON—MATH 350—CLASSROOM WORKSHEET 35
The Party Problem—Ramsey Theory

Assume that *knowing someone* is a reflexive relation: that is, if A knows B then B knows A.

The **Party Problem** is the problem of determining the fewest number of people at a party so that there are either k people who mutually know each other—or k people none of which knows any of the others.

We can *represent* the people at a party with and their relationships with a *graph*: dots, a solid line between a pair of dots if these people know each other, and a dotted line if they do not. So there is a line between each pair of dots.

Let $R(s,d)$ be the smallest number so that *any* party with this many people either contains s people who all know each other (all solid lines—an s-clique) or d people none of which knows another (all dotted lines, a d-clique).

We will show that $R(s,d)$ *exists*; in particular, for any s and any d there is *some number* so that if a party contains that many people, there *must* be either s who all know each other or d people none of which knows another.

To show that $R(s,d) = n$ you must do 2 things:

1. Find a graph with $n-1$ dots and *no* s-clique and no d-clique.
2. Show that *any* graph with n dots *must* have an s-clique or a d-clique.

We showed: $R(2,2) = 2$, $R(2,3) = R(3,2) = 3$, $R(3,3) = 6$, $R(2,4) = 4$, and $R(3,4) \leq 10$.

Problems

1. Find $R(2,k)$, where k is an integer greater than 1.

Claim: $R(s,d) \leq R(s-1,d) + R(s,d-1)$. We will prove this by induction.
Note: a corollary is that $R(s,d)$ exists!

2. What does the above claim say about $R(4,4)$?

So this gives existence and an *upper bound* for $R(k,k)$. Now we'll find a *lower bound* for $R(k,k)$. (We may not be able to find a formula for $R(k,k)$ but finding and improving upper and lower bounds can get us closer).

3. Given k people at a party, how many pairs do they form?

4. If a party of n people is chosen randomly and it is equally likely for any pair of people to know each other, and k of these people are selected, what is the probability that all k people know each other?