Last name _____

First name _____

LARSON—MATH 350—CLASSROOM WORKSHEET 17
Ballot Sequences & Bracketings

Organizational Notes

1. A Zoom recording link and class notes will be sent out after each class.

2. Remember to send your answers to the classroom worksheets. Title your email with enough to help me record your "participation".

Review

1. Show that every plane tree with $n+1$ nodes can be associated to a unique binary tree on n nodes (this implicitly defines a one-to-one function from the set of the plane trees to the set of the binary trees trees).

2. What is a *ballot sequence*?

Counting Ballot Sequences (Sec. 1.5)

1. Find all ballot sequences with 0 1's and 0 -1's ($n = 0$ case, length $= 2n = 0$).

2. Find all ballot sequences with 1 1's and 1 -1's ($n = 1$ case, length $= 2n = 2$).

3. Find all ballot sequences with 2 1's and 2 -1's ($n = 2$ case, length $= 2n = 4$).

4. Find all ballot sequences with 3 1's and 3 -1's ($n = 3$ case, length $= 2n = 6$).

5. Find all ballot sequences with 4 1's and 4 -1's ($n = 4$ case, length $= 2n = 8$).

6. Can you conjecture the number of ballot sequences with length $2n$????

7. How can you *prove* this conjecture?

Parenthesization/Bracketing

8. What is a *bracketing* of a string of $n+1$ x's?

9. Find all bracketings of strings of 1 x's ($n = 0$ left parentheses, and $n = 0$ right parentheses).

10. Find all bracketings of strings of 2 x's ($n = 1$ left parentheses, and $n = 1$ right parentheses).

11. Find all bracketings of strings of 3 x's ($n = 2$ left parentheses, and $n = 2$ right parentheses).

12. Find all bracketings of strings of 4 x's ($n = 3$ left parentheses, and $n = 3$ right parentheses).

13. Find all bracketings of strings of 5 x's ($n = 4$ left parentheses, and $n = 4$ right parentheses).