Last name _____

First name _____

LARSON—MATH 350—CLASSROOM WORKSHEET 15
Bijective Proofs, Binary Trees & Plane Trees

Organizational Notes

1. A Zoom recording link and class notes will be sent out after each class.

2. Remember to send your answers to the classroom worksheets. Title your email with enough to help me record your "participation".

3. Homework 4 (the Test Review) is due Monday (@11:59).

4. Test 1 is Tuesday, Oct. 13.

Review

1. (**Counting Divisors**.) How many divisors does an integer $n = p_1^{n_1} \cdot p_2^{n_2} \cdot \ldots \cdot p_k^{n_k}$ have.

Counting Binary Trees (Sec. 1.5)

1. Show that every triangulated polygon with $n+2$ sides can be associated to a unique binary tree on n nodes (this implicitly defines a one-to-one function from the set of these polygons to the set of these trees).

2. What can you conclude?

Plane Trees (Sec. 1.5)

3. What is a *plane tree*?

4. How many plane trees are there with 2 nodes?

5. How many plane trees are there with 3 nodes?

6. How many plane trees are there with 4 nodes?

7. How many plane trees are there with 5 nodes?

8. Can you *conjecture* how many plane trees there are with $n+1$ nodes?

9. Can you *prove* your formula?

10. Show that every plane tree with $n+1$ nodes can be associated to a unique binary tree on n nodes (this implicitly defines a one-to-one function from the set of the plane trees to the set of the binary trees trees).

    (**From our text**). "Given a plane tree P with $n+1$ vertices, first remove the root vertex and all incident edges. Then remove every edge that is not the leftmost edge from a vertex. The remaining edges are the left edges in a binary tree B. whose root is the leftmost child of the root of P. Now draw edges from each child w of a vertex v of P to the next child (the one immediately to the right of w) of v (if such a child exists). These horizontal edges are the right edges of B. The steps can be reversed to recover P from B, so the map $P \to B$ gives the desired bijection."