Last name ______________________
First name ______________________

LARSON—MATH 350—CLASSROOM WORKSHEET 02
Multiplication Principle & 1st formula for the Catalan numbers

Organizational Notes

1. A Zoom recording link and class notes will be sent out after each class.

2. Remember to send your answers to the classroom worksheets. Title your email with enough to help me record your participation on Blackboard, something like “Math 350, 8/18 classroom worksheet attached”.

3. Watch for Homework #1 (h01) worksheet in your email.

4. Read ahead in Chp.1 of Stanley’s book—see if you can figure out the first formula for the Catalan numbers on p.3.

Review

1. (Notation). What are $\mathbb{N}$, $\mathbb{P}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$?

2. (Notation). What is $[n]$ (for $n \in \mathbb{P}$)?

3. (Notation). What is $\#S$ (for a finite set $S$)?

4. What is the convex polygon $\mathcal{P}_{n+2}$?

5. What is a triangulation of a convex polygon?

New Material

1. What is $C_n$, the $n^{th}$ Catalan number?
2. Find $C_1$.
3. Find $C_2$.
4. Find $C_3$.
5. Find $C_4$.
6. Find $C_5$. 
7. There are 2 drink choices, and 3 entree choices. How many ways are there to choose 1 drink and 1 entree?

8. What is the Multiplication Principle?

9. There are 2 drink choices, 3 entree choices, and 4 desert choices at a restaurant. How many ways are there to choose 1 drink, 1 entree and 1 desert?

10. How can the Multiplication Principle be extended to more than two choices?

11. There are 6 guests at a birthday party. Each shakes the hand of every other (exactly once). How many handshakes occur?

12. Alice sits at a fixed spot of a round table. There are 5 other chairs around the table. How many ways are there to seat the 5 other guests?

13. How many ways are there to seat the 6 guests if Alice’s spot isn’t fixed?

14. A lottery ticket consists of a choice of 5 different numbers from 90 possible numbers. How many different choices are possible?

15. Consider one side of a triangulated polygon $P_{n+2}$; note that’s its endpoints must both connect to the same vertex (the endpoints and the vertex form a triangle in the triangulation). How many vertices could those endpoint connect to?

16. Consider again one side of a triangulated polygon $P_{n+2}$. If that edge is removed two new (triangulated) polygons are formed. How many ways are there to triangulate each of these polygons?

17. Use the following ingredients to determine the number $C_n$:
   (a) Any side of a triangulated polygon $P_{n+2}$ must appear in some triangle of any triangulation.
   (b) There are $n$ possibilities for the third vertex of the triangle with this side.
   (c) Removing this edge leaves two smaller polygons $C_a$ and $C_b$ and the number of these triangulations is $C_a \cdot C_b$.
   (d) The number $C_n$ can be found by adding up these terms for each case.

18. Now check our theoretical formula against our direct counts. If they are not the same then something is wrong!